



A CRITICAL-MACHINE BASED HEURISTIC FOR HFS BATCH SCHEDULING

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ABSTRACT

The supply chain management encourages many entrepreneurs to produce specific product with world class quality. So variety jobs to be handled with minor variations in design or dimensions to worldwide customers. According to group technology, the operation based grouping of component types for manufacturing by batch. In which the due date based selection of job types for processing with priority. The manufacturing environment often changes by including standby or rental machine to meet delivery schedule. The flexible or hybrid flow shop is flow shop with parallel machine(s) at some stages. In this work an industrial problem is considered. The manufacturing configuration is two stage hybrid flowshop. The common due date based job types from 11 to 15 were considered at five levels. Applied mathematical model presented and the same was used to develop the simulation model. The simulation model was used for lot sizing and validation of lot streaming heuristic. The developed heuristics out performed and develop more than one best schedule. Hence the grouping difficulties in practice were minimized.

Keywords: Hybrid Flow shop, group Scheduling, Critical Machine, mathematical model, simulation, lot sizing, group schedule, makespan

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1. INTRODUCTION

Hybrid Flow shop (HFS) scheduling is a combinatorial in nature. The two stage HFS is very renowned configuration in many industries. The batch scheduling can be classified as Level I and Level II. The Level II problem which involves, lot forming, lot sizing and lot scheduling (group scheduling). The Level I in which schedule the jobs within the lot. The Hybrid Flow Shop (HFS) environment characterized by production flow shop with single/multiple parallel machines added in each production stage, As HFS problems NP hard in nature they can solve by use of polynomial with some special properties and precedence relationships or heuristic method [1]. The two stage hybrid Flowshop with two dedicated machines in the First stage and single machine in the stage two. Similar environment is considered for scheduling by JaehwanYang [2]. The author developed a simple heuristics by use of greedy along with Johnson's rule for minimizing the makespan. Jianming Dong et al [3] proposed an approximation algorithm to solve the problem with ratio of machines in the first and second stage of two stage HFS. A fully polynomial time approximation scheme was proposed [4] to solve the two stage HFS problem with flexible machining at first stage and limited buffer constraints. [5] Suggested six versions of hybrid particle swarm optimization to solve the two stage three machine Flowshop scheduling problem. Such applications can be found in assembly sections of fire engine and PC (Personal computers. The above cases are belongs to level I scheduling cases. [6] suggested a mathematical model for solving lot-streaming HFS batch scheduling problem (Level II) in which they considered priority rule with shortest weighting time and concurrent arrival of jobs. The dual objectives of minimizing the total weighted tardiness and weighted sum of the total weighted completion time were fulfilled in a batch scheduling of two stage HFS with sequence- and machine-dependent family setup times the by using robust meta-heuristics[7]. Vivek et al [8] suggested mathematical model and heuristic for 2 stage HFS batch scheduling problem. The Critical machine base scheduling is another approach to solve scheduling problem which gives priority to jobs at critical machine while scheduling [9]. The critical machine based was reviews for Level I problems [10] and Level I and Level II problems reported separately. This paper deals with the two stage hybrid flowshop batch scheduling of lots of fifteen different job types. The Figure 1 shows the configuration of the two stage Hybrid flow shop problem. The first stage contains two identical processors in parallel with the name of $M_{1(a)}$ and $M_{1(b)}$. The Second stage has only one machine. The second stage machine often encountered as critical based on the processing times of the jobs. The jobs queue status to be checked hence simulation is preferred in this paper as suggested by [11].

2. PROBLEM FORMULATION

2.1. Assumptions

The following assumptions are made

- i. Job flow unidirectional.
- ii. Preemption is prohibited.
- iii. Loading, unloading and Setup times are included in the processing time.

- iv. Initially assumed unlimited intermittent storage capacity.
- v. Jobs available at zero time at the first stage.
- vi. There is no interruption i.e., facilities available continuously.
- vii. Splitting of the individual sub - lot is not permitted for forming the lot. But sub lots may be grouped to form lots with all possible ways.

A lot can be subsequently considered for processing after the completion of the last job of the last lot at all the stages

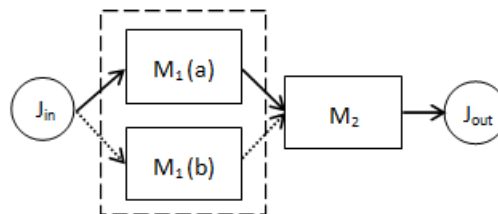


Figure 1 Two stage hybrid flowshop

2.2. Parameters

J_i = the job J_i ($J_i=1, 2, \dots, 200$) of provided i^{th} ($i = 1, 2, \dots, k$), sub-lot,

k = Number of sub-lots here cases are $k(k=6, 7, \dots, 10)$

G_s = Group schedule

$J_i \in J$,

J = Total number of jobs to be scheduled,

$(P_m)_{J_i}$ = Processing times of job,

J_i of sub-lot k at m^{th} stage,

$m = 1, 2, (m = 2, M^{(1)} = 1, M^{(2)} = 2)$,

T = Total number of time units in scheduling jobs,

$(C_m)_{J_i}$ = Completion time of job J of sub-lot i at the stage m ,

It is a time point $(C_m)_{J_i} = t$ means that the operation completes at the end of the time unit t .

2.3. Decision variables

$$(\delta_m)_{J_i} t = \left\{ \begin{array}{ll} 1 & \text{if } J_i \text{ is processed at the stage } m \text{ in the time unit } t; \\ 0 & \text{otherwise} \end{array} \right\}$$

with the above notation, the hybrid flow shop problem (HFSP) under the consideration can be formulated as follows.

The $(C_2)_{J_i}$ in the model is the final completion time of the male job J of type i , C_2 after processed at required at the stage 2, in other words it is the completion time of a job to the sub-lot i .

2.4. Mathematical Model

The lot may be formed by combining of sub-lots or sub lot alone as per lot streaming strategy; it is chosen based on the available sub-lots. Lot completion time is the completion time of the last job of that lot. Makespan is the completion time of the last job of the last lot in the group schedule (G_s). The objective is minimizing makespan and the objective function is:

$$\begin{aligned}
 & \text{Minimize } \sum_{i=1}^k \sum_{J_i=1}^n (C_2)_{J_i} && 1 \\
 & \text{Where varies five levels as } k= 11,12,\dots,15 \text{ and } n = 200 \\
 & \text{Subject to} \\
 & (C_2)_{J_i} \leq (C_2)_{J_i} + 1 - (P_2)_{J_i} \quad J_i (J_i=1,2,\dots,n); J_i \in J; && 2 \\
 & \sum_{t=1}^{T_m} (\delta_2)_{J_i} t = (P_2)_{J_i} \quad ; t = 1,2,\dots,T && 3 \\
 & t * (\delta_2)_{J_i} t \leq (C_2)_{J_i} && 4 \\
 & (C_2)_{J_i} - (P_2)_{J_i} + 1 \leq t + T(1 - (\delta_2)_{J_i} t) && 5 \\
 & \sum_{J_i=1}^n (\delta_2)_{J_i} t \leq M^{(s)} \quad ; \quad M^{(1)} = 1, M^{(2)} = 2 && 6 \\
 & (\delta_2)_{J_i} t \in \{0,1\} && 7 \\
 & (C_2)_{J_i} \in \{1,2,\dots,T\} && 8
 \end{aligned}$$

In this mathematical model, the objective is to find the optimal group scheduling that minimizes the makespan with consideration of critical machine, at the same time it should satisfy the listed constraints in the equation (2) to the equation (8). The equation (2) is precedence constraint, *i.e.*, an operation cannot be started until the operation of the same job at its preceding stage is completed. The equations (3) – (5) defines the time intervals for which a job is processed on a machine at a stage. The equation (6) defines the machine constraint at each stage *i.e.*, machine requirements are satisfied with the number of available machines at that time. The equation (7) & (8) provides the time range of the variables.

2.5. Makespan Computation

Mathematical representation of computation of makespan for the example group schedule $\{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_5 \& J_6)\}$ is

Mathematical representation of computation of makespan for a sample group schedule $\{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_5 \& J_6)\}$ is

$$\begin{aligned}
 C_j = & \max \left\{ \sum_{j_i=1}^{200} (C_2)_{j_1}, \sum_{j_i=1}^{200} (C_2)_{j_2} \right\} + \max \left\{ \sum_{j_i=1}^{200} (C_2)_{j_3}, \sum_{j_i=1}^{200} (C_2)_{j_4} \right\} \\
 & + \max \left\{ \sum_{j_i=1}^{200} (C_2)_{j_5}, \sum_{j_i=1}^{200} (C_2)_{j_6} \right\}
 \end{aligned}$$

2.6. Problem Description

The computational experiments were carried out with real world environment observations *i.e.*, the automobile spare production industry. The industry has worldwide customers and

launched huge variety as per demand. Hence the job varieties are included according to the delivery schedule. As on date there are possibilities to have six to ten job varieties with same lot size was estimated. The maximum number of lots considered depends up on the delivery schedule but as per history of production minimum 6 lots considered. Hence this computation experiments consider at different level that is the job variety levels from six to ten k (k=11,12,...,15). The manufacturing environment is classified as $m = 2, M^{(1)} = 2, M^{(2)} = 1$; hybrid flowshop Problem. That is the jobs are processed at two stages in the manufacturing shop in which at first stage has two processors/ machines and the next stage contains one machine / processor. Total number of job variety involved is 15. A sub-lot possess similar jobs of 200 with same processing time and processing sequence. Hence, the maximum number jobs to be processed are 3000 jobs with 15 varieties. The job processing times are not similar for all jobs. The job variety and their processing time sequence are furnished in Table 1.

2.7. Critical Machine

The second stage of the Hybrid Flowshop has only one machine and the same is engaging to process the jobs which processed by both machines at the first stage. If the job scheduled is more (if number of sub lot is more in the lot means available job at first stage will be more) at first stage the two machines may be engaged for processing else the first machine is only preferred for processing of all jobs in the first stage. The second stage may be bottle neck when the output of first stage is greater than the second stage. Hence the second stage machine is called Critical machine. The critical machine based scheduling is one, which consider either the capacity or some other constraint related to the critical machine.

3. SIMULATION MODELLING

The mathematical model used for creating the simulation model. The Extend v6 software employed to create the model of the production environment. The model is verified and validated by real world data. The model has facilitated to collect statistics of queue status like maximum waits, average waits, maximum length and average length, machine utilization status and flow statistics like mean flow time and completion time.

4. HEURISTIC SOLUTIONS

Step 1. As the inherent sub lot size is same as well as the processing time and processing sequence of jobs within the inherent sub lot is also same. One job per variety is to be considered for computation of Total Processing time T_p . The total processing time T_p of all job varieties k (k=1,2,...,15) can be mathematically written as

$$T_p = \sum_{i=1}^k \sum_{m=1}^2 P_{mji} \quad \forall T; T_p \in T$$

- a. Compute total processing time for each job variety. t_p .
- b. The sum of the processing time for each job variety is T_p .

Step 2: Computation of desired number of lots (N_l)

Total number of sub-lots involved (k).

Case 1. For almost Uniform lot size, the desired number of lots $N_l = \left(\frac{k}{S_{max}} \right)$

If round up to whole number and consider the same as N_l

Case 2. For Non-Uniform lot sizes or customized lot size, the desired number of lots (N_l) to be identified manually and consider.

Step 3: Compute the Average processing time

$$m_p = \frac{T_p}{N_l}$$

Step 4: lot formation

Form the N_l lots with lot size of S_{max} and S_{min}

Step 5: Group scheduling

- a. Check deviations $d_i = t_{pi} - m_p$ where $i (i=1,2,\dots, N_l)$
- b. compute net deviation $d = \sum_{i=1}^{N_l} d_i$
- c. Stop if $d = 0$ else Go to step 4b
- d. The schedule is optimal group schedule

5. LOT SIZING

In this part of study only first 6 sub-lots are considered with minimum lot size as one sub-lot and the maximum sub-lots per lot is six. There are eleven lost streaming strategies (LSS) derived. Heuristic used to develop the best schedule for each lot streaming strategy. The lot sizes are considered both uniform and non-uniform. The LSS and its best schedule were presented in the Table 2. The LSS wise best group schedule is simulated and its makespan shown in table 2 and compared in Figure 2.

Table 1 processing times of Jobs

| Ji | Stage-1 (min) | Stage-2 (min) | Ji | Stage-1 (min) | Stage-2 (min) |
|----|---------------|---------------|----|---------------|---------------|
| 1 | 5.42 | 1.5 | 9 | 3.92 | 5.22 |
| 2 | 2.7 | 4.94 | 10 | 3.42 | 5.38 |
| 3 | 2.86 | 2.9 | 11 | 4.3 | 3.34 |
| 4 | 4.22 | 3.94 | 12 | 2.9 | 5.82 |
| 5 | 5.86 | 1.94 | 13 | 5.3 | 3.2 |
| 6 | 4.53 | 3.66 | 14 | 4.62 | 3.15 |
| 7 | 1.52 | 4.57 | 15 | 3.32 | 4.7 |
| 8 | 3.02 | 5.41 | | | |

Table 2 details of lot streaming strategies and their optimal makespan

| LSS* No. | Lot forming strategy [No. of Sub-lot(s)] | No. of Lot | Optimal Group Schedule (G_o) | Make span in Minutes |
|----------|--|------------|---|----------------------|
| 1 | 6 | 1 | { $L_1(j_1, j_2, j_3, j_4, j_5 \& j_6)$ } | 3781 |
| 2 | 5 & 1 | 2 | { $L_1(j_1, j_2, j_4, j_5 \& j_6) - L_2(j_3)$ } | 3784 |
| 3 | 4 & 2 | 2 | { $L_1(j_1, j_2, j_3 \& j_6) - L_2(j_4 \& j_5)$ } | 3783 |
| 4 | 3 & 3 | 2 | { $L_1(j_1, j_3 \& j_6) - L_2(j_2, j_4 \& j_5)$ } | 3782 |
| 5 | 2, 2 & 2 | 3 | { $L_1(j_1 \& j_6) - L_2(j_2 \& j_3) - L_3(j_4 \& j_5)$ } | 3788 |
| 6 | 3, 2 & 1 | 3 | { $L_1(j_1, j_3 \& j_6) - L_2(j_4 \& j_5) - L_3(j_2)$ } | 3786 |
| 7 | 4, 1 & 1 | 3 | { $L_1(j_1, j_2, j_3 \& j_5) - L_2(j_4) - L_3(j_6)$ } | 3983 |
| 8 | 2, 2, 1 & 1 | 4 | { $L_1(j_1 \& j_4) - L_2(j_5 \& j_6) - L_3(j_2) - L_4(j_3)$ } | 3791 |
| 9 | 3, 1, 1 & 1 | 4 | { $L_1(j_1, j_2 \& j_3) - L_2(j_4) - L_3(j_5) - L_4(j_6)$ } | 3990 |
| 10 | 2, 1, 1, 1 & 1 | 5 | { $L_1(j_1 \& j_3) - L_2(j_2) - L_3(j_4) - L_4(j_5) - L_5(j_6)$ } | 3993 |
| 11 | 1, 1, 1, 1 & 1 | 6 | { $L_1(j_1) - L_2(j_3) - L_3(j_2) - L_4(j_4) - L_5(j_5) - L_6(j_6)$ } | 4238 |

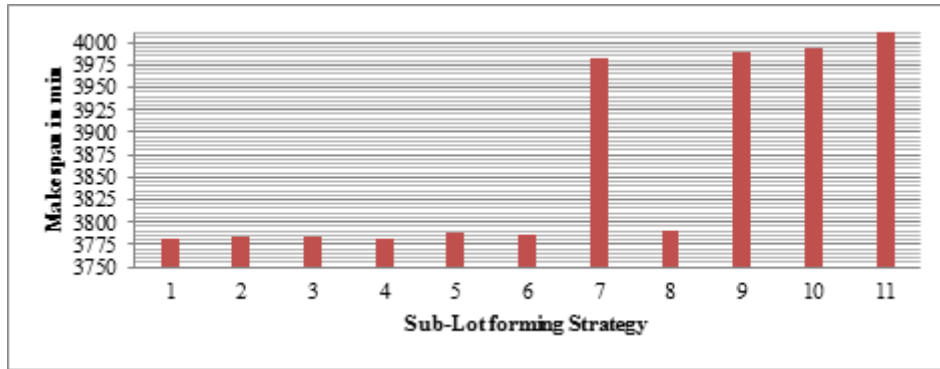


Figure 2 Lot streaming strategies and their optimal makespan

Table 3 Best lot streaming strategies & Average Queue status at Critical machine

| Best LSS | Avg. Length | Ave. Waits |
|----------|-------------|------------|
| 1 | 22 | 532 |
| 2 | 11 | 326 |
| 3 | 15 | 535 |
| 4 | 8 | 395 |
| 5 | 2 | 135 |
| 6 | 8 | 395 |
| 8 | 3 | 213 |

From the figure 2 & Table 2, it is reveals that the strategies 1,2,3,4,5,6 and 8 gives minimal makespan with maximum variation of 10 minutes. So the above said strategies can be adopted complete the order in time. But the increase of number of sub-lots increases the strategies. Hence the optimization of strategy is primary issue. The observations of the queue status at critical machine for best strategies are presented in Table 3. From the Table 3 it is cleared that the best strategy is 5th one. Because 5th strategy yields 3788 minutes makespan with minimal average queue length 2 jobs and average job waiting time is 135 minutes.

6. NUMERICAL EXAMPLE

As per case study the maximum job varieties (k) are 15 i.e., 15 sub-lots, But here 10 sub-lots as maximum as considered for illustrating the heuristic. The uniform sub-lot size is 200 jobs, the processing time listed in Table 1. The optimized lot size details are: maximum 2 sub-lots (S_{max}) and minimum 1 sub-lot (S_{min}).

Step 1: Refer Table 1 for Computation of (T_p)

$$T_p = \sum_{i=1}^k \sum_{m=1}^2 P_{mji} \quad \forall T; T_p \in T \quad T_p = 76.93 \text{ minutes}$$

Step 2: Computation of desired number of lots (Nl)

Total number of sub-lots involved (k) is 15. The lot size is almost uniform and odd number of total sub-lots,

$$\text{Hence the desired number of lots } N_i = \left(\frac{k}{S_{max}} \right) = \frac{10}{2} = 5$$

$$\text{Step 3: Compute the mean processing time } m_p = \frac{T_p}{N_i} = \frac{76.93}{5} = 15.386$$

Step 4: Sub-lot formation

- Computation of total processing time (tp) by adding all processing time in Table 1.
- Form the Nl lots with lot size of S_{max} and S_{min}
- Check deviations $d_i = t_{pi} - m_p$ where i (i=1,2,..., Nl)

- d. calculate net déviation $d = \sum_{i=1}^{N_i} d_i$
- e. Stop if $d = 0$ else Go to step 4b

$$G_s = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_6 \& J_9) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_5)\}$$

$$d = -0.826 - 1.466 + 1.944 - 0.866 + 1.214 = 0.000000000000000710543$$

Hence the G_s is the optimal group schedule. Computation of makespan is sum of the sub group completion time. Hence

$$C_j = 1291 + 1371 + 1780 + 1898 + 1467 = 7807 \text{ minutes.}$$

7. BEST SCHEDULES

The developed heuristic was applied to various maximum sub-lots involved cases i.e., $k=6, 7, \dots, 10$. At each case more than one optimal schedules and its equivalent were found. Here below the case wise they are presented. $C_{jL1}, C_{jL2}, C_{jL3}, C_{jL4}$ and C_{jL5} are the lot completion time in minutes of lot 1, lot 2, lot 3, lot 4 and lot 5 respectively.

7.1. Case -I

In this case maximum sub-lots considered are eleven. The best group schedules are furnished below and their individual lot completion times ($C_{jL1}, C_{jL2}, C_{jL3}, C_{jL4}, C_{jL5}$ & C_{jL6}) and makespan (C_j) can be found in Table 4.

Table 4 Lot completion times and makespan for case-I best schedules

| Gs | C _{jL1} | C _{jL2} | C _{jL3} | C _{jL4} | C _{jL5} | C _{jL6} | C _j |
|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|----------------|
| G ₁ | 1291 | 1371 | 1436 | 1898 | 1811 | 672 | 8479 |
| G ₂ | 1291 | 1371 | 1780 | 1898 | 1467 | 672 | 8479 |
| G ₃ | 1291 | 1371 | 1060 | 1898 | 2123 | 737 | 8480 |
| G ₄ | 1291 | 1371 | 1125 | 1898 | 1744 | 1048 | 8477 |
| G ₅ | 1291 | 1371 | 1125 | 1898 | 1713 | 1079 | 8477 |
| G ₆ | 1291 | 1251 | 1125 | 1898 | 2123 | 793 | 8481 |
| G ₇ | 1291 | 1457 | 1125 | 1898 | 2123 | 583 | 8477 |
| G ₈ | 1291 | 1315 | 1181 | 1898 | 2123 | 672 | 8480 |
| G ₉ | 883 | 1181 | 1780 | 1898 | 2067 | 672 | 8481 |
| G ₁₀ | 883 | 1779 | 1125 | 1898 | 2123 | 672 | 8480 |
| G ₁₁ | 883 | 1723 | 1181 | 1898 | 2123 | 672 | 8480 |
| G ₁₂ | 1093 | 1571 | 1125 | 1898 | 2123 | 672 | 8482 |
| G ₁₃ | 1037 | 1571 | 1181 | 1898 | 2123 | 672 | 8482 |
| G ₁₄ | 1379 | 1571 | 1181 | 1780 | 1898 | 672 | 8481 |
| G ₁₅ | 975 | 1371 | 1125 | 1898 | 2123 | 991 | 8483 |

$$G_1 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_5 \& J_9) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_6) - L_6(J_{11})\}$$

$$G_2 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_6 \& J_9) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_5) - L_6(J_{11})\}$$

$$G_3 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_{11} \& J_5) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_6)\}$$

$$G_4 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_{11}) - L_6(J_9)\}$$

$$G_5 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{11} \& J_9) - L_6(J_{10})\}$$

$$G_6 = \{L_1(J_1 \& J_2) - L_2(J_{11} \& J_3) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_4)\}$$

$$G_7 = \{L_1(J_1 \& J_2) - L_2(J_{11} \& J_4) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_3)\}$$

$$G_8 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_6) - L_3(J_4 \& J_5) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_{11})\}$$

$$G_9 = \{L_1(J_1 \& J_3) - L_2(J_4 \& J_5) - L_3(J_6 \& J_9) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_2) - L_6(J_{11})\}$$

- G10= {L1(J1 & J3) - L2(J2 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J11)}
- G11= {L1(J1 & J3) - L2(J2 & J6) – L3(J4 & J5) – L4(J7 & J8) – L5(J10 & J9) – L6(J11)}
- G12= {L1(J1 & J4) - L2(J2 & J3) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J11)}
- G13= {L1(J1 & J6) - L2(J2 & J3) – L3(J4 & J5) – L4(J7 & J8) – L5(J10 & J9) – L6(J11)}
- G14= {L1(J10 & J1) - L2(J2 & J3) – L3(J4 & J5) – L4(J6 & J9) – L5(J7 & J8) – L6(J11)}
- G15= {L1(J11 & J1) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J2)}

7.2. Case -II

In this case maximum twelve sub-lots are considered. The best group schedules are listed here below. The individual lot completion times (C_{jL1} , C_{jL2} , C_{jL3} & C_{jL4}) and makespan (C_j) for each schedule may be found in Table 5.

- G1= {L1(J1 & J2) - L2(J11 & J3) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J4 & J12)}
- G2= {L1(J1 & J2) - L2(J11 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J3 & J12)}
- G3= {L1(J1 & J2) - L2(J3 & J4) – L3(J11 & J5) – L4(J7 & J8) – L5(J10 & J9) – L6(J6 & J12)}
- G4= {L1(J1 & J2) - L2(J3 & J4) – L3(J11 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J5 & J12)}
- G5= {L1(J1 & J2) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J11) – L6(J9 & J12)}
- G6= {L1(J1 & J2) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J11 & J9) – L6(J10 & J12)}
- G7= {L1(J1 & J2) - L2(J3 & J4) – L3(J5 & J9) – L4(J7 & J8) – L5(J10 & J6) – L6(J11 & J12)}
- G8= {L1(J1 & J2) - L2(J3 & J4) – L3(J6 & J9) – L4(J7 & J8) – L5(J10 & J5) – L6(J11 & J12)}
- G9= {L1(J1 & J2) - L2(J3 & J6) – L3(J4 & J5) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12)}
- G10= {L1(J1 & J3) - L2(J2 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12)}
- G11= {L1(J1 & J3) - L2(J2 & J6) – L3(J4 & J5) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12)}
- G12= {L1(J1 & J4) - L2(J2 & J3) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12)}
- G13= {L1(J1 & J6) - L2(J2 & J3) – L3(J4 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12)}
- G14= {L1(J10 & J1) - L2(J2 & J3) – L3(J4 & J5) – L4(J7 & J8) – L5(J6 & J9) – L6(J11 & J12)}
- G15= {L1(J11 & J2) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J1 & J12)}
- G16= {L1(J11 & J1) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J2 & J12)}

Table 5 Lot completion times and makespan for case-II best schedules

| Gs | C_{iL1} | C_{iL2} | C_{iL3} | C_{iL4} | C_{iL5} | C_{iL6} | C_i |
|-----------------|-----------|-----------|-----------|-----------|-----------|-----------|-------|
| G ₁ | 1291 | 1251 | 1125 | 1898 | 2123 | 1955 | 9643 |
| G ₂ | 1291 | 1457 | 1125 | 1898 | 2123 | 1747 | 9641 |
| G ₃ | 1291 | 1371 | 1060 | 1898 | 2123 | 1899 | 9642 |
| G ₄ | 1291 | 1371 | 1404 | 1898 | 2123 | 1555 | 9642 |
| G ₅ | 1291 | 1371 | 1125 | 1898 | 1744 | 2211 | 9640 |
| G ₆ | 1291 | 1371 | 1125 | 1898 | 1713 | 2243 | 9641 |
| G ₇ | 1291 | 1371 | 1436 | 1898 | 1811 | 1835 | 9642 |
| G ₈ | 1291 | 1371 | 1780 | 1898 | 1467 | 1835 | 9642 |
| G ₉ | 1291 | 1315 | 1181 | 1898 | 2123 | 1835 | 9643 |
| G ₁₀ | 883 | 1779 | 1125 | 1898 | 2123 | 1835 | 9643 |

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| | | | | | | | |
|-----------------|------|------|------|------|------|------|------|
| G ₁₁ | 883 | 1723 | 1181 | 1898 | 2123 | 1835 | 9643 |
| G ₁₂ | 1093 | 1571 | 1125 | 1898 | 2123 | 1835 | 9645 |
| G ₁₃ | 1037 | 1571 | 1181 | 1898 | 2123 | 1835 | 9645 |
| G ₁₄ | 1379 | 1571 | 1181 | 1898 | 1780 | 1835 | 9644 |
| G ₁₅ | 1659 | 1371 | 1125 | 1898 | 2123 | 1467 | 9643 |
| G ₁₆ | 975 | 1371 | 1125 | 1898 | 2123 | 2155 | 9647 |

7.3. Case -III

In this case maximum sub-lots are thirteen. The best group schedules for this case are listed below. The each lot completion time of the each best group schedule and its makespan (C_j) shown in Table 6.

Table 6 Lot completion times and makespan for case-III best schedules

| Gs | C _{iL1} | C _{iL2} | C _{iL3} | C _{iL4} | C _{iL5} | C _{iL6} | C _{iL7} | C _i |
|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|----------------|
| G ₁ | 1291 | 1429 | 1125 | 1898 | 2123 | 1835 | 583 | 10284 |
| G ₂ | 1291 | 1371 | 1125 | 1898 | 1685 | 1835 | 1079 | 10284 |
| G ₃ | 1291 | 1371 | 1125 | 1898 | 1744 | 2211 | 645 | 10285 |
| G ₄ | 1291 | 1457 | 1125 | 1898 | 2123 | 1747 | 645 | 10286 |
| G ₅ | 1291 | 1371 | 1125 | 1898 | 1713 | 2243 | 645 | 10286 |
| G ₆ | 1291 | 1251 | 1125 | 1898 | 2123 | 1955 | 645 | 10288 |
| G ₇ | 1291 | 1371 | 1060 | 1898 | 2123 | 1899 | 645 | 10287 |
| G ₈ | 1291 | 1371 | 1404 | 1898 | 2123 | 1555 | 645 | 10287 |
| G ₉ | 1291 | 1371 | 1436 | 1898 | 1811 | 1835 | 645 | 10287 |
| G ₁₀ | 1291 | 1371 | 1125 | 1898 | 1719 | 1835 | 1048 | 10287 |
| G ₁₁ | 1291 | 1371 | 1125 | 1898 | 2123 | 1807 | 672 | 10287 |
| G ₁₂ | 1291 | 1371 | 1125 | 1898 | 2123 | 1312 | 1167 | 10287 |
| G ₁₃ | 1291 | 1315 | 1181 | 1898 | 2123 | 1835 | 645 | 10288 |
| G ₁₄ | 1291 | 1223 | 1125 | 1898 | 2123 | 1835 | 793 | 10288 |
| G ₁₅ | 883 | 1779 | 1125 | 1898 | 2123 | 1835 | 645 | 10288 |
| G ₁₆ | 883 | 1723 | 1181 | 1898 | 2123 | 1835 | 645 | 10288 |
| G ₁₇ | 1659 | 1371 | 1125 | 1898 | 2123 | 1467 | 645 | 10288 |
| G ₁₈ | 883 | 1181 | 1780 | 1898 | 2067 | 1835 | 645 | 10289 |
| G ₁₉ | 1571 | 1181 | 1780 | 1898 | 1379 | 1835 | 645 | 10289 |
| G ₂₀ | 1093 | 1571 | 1125 | 1898 | 2123 | 1835 | 645 | 10290 |
| G ₂₁ | 1037 | 1571 | 1181 | 1898 | 2123 | 1835 | 645 | 10290 |
| G ₂₂ | 975 | 1371 | 1125 | 1898 | 2123 | 2155 | 645 | 10292 |

G1= {L1(J1 & J2) - L2(J13 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J11 & J12) - L7(J3) }

G2= {L1(J1 & J2) - L2(J3 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J13 & J9) - L6(J11 & J12) - L7(J11) }

G3= {L1(J1 & J2) - L2(J3 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J11) - L6(J9 & J12) - L7(J13) }

G4= {L1(J1 & J2) - L2(J11 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J3 & J12) - L7(J13) }

G5= {L1(J1 & J2) - L2(J3 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J11 & J9) - L6(J10 & J12) - L7(J13) }

G6= {L1(J1 & J2) - L2(J11 & J3) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J4 & J12) - L7(J13) }

G7= {L1(J1 & J2) - L2(J3 & J4) - L3(J11 & J5) - L4(J7 & J8) - L5(J10 & J9) - L6(J6 & J12) - L7(J13) }

$$G8= \{L1(J1 \& J2) - L2(J3 \& J4) - L3(J11 \& J6) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J5 \& J12) - L7(J13) \}$$

$$G9= \{L1(J1 \& J2) - L2(J3 \& J4) - L3(J5 \& J9) - L4(J7 \& J8) - L5(J10 \& J6) - L6(J11 \& J12) - L7(J13) \}$$

$$G10= \{L1(J1 \& J2) - L2(J3 \& J4) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J13) - L6(J11 \& J12) - L7(J9) \}$$

$$G11= \{L1(J1 \& J2) - L2(J3 \& J4) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J13 \& J12) - L7(J11) \}$$

$$G12= \{L1(J1 \& J2) - L2(J3 \& J4) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J13 \& J11) - L7(J12) \}$$

$$G13= \{L1(J1 \& J2) - L2(J3 \& J6) - L3(J4 \& J5) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J11 \& J12) - L7(J13) \}$$

$$G14= \{L1(J1 \& J2) - L2(J13 \& J3) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J11 \& J12) - L7(J4) \}$$

$$G15= \{L1(J1 \& J3) - L2(J2 \& J4) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J11 \& J12) - L7(J13) \}$$

$$G16= \{L1(J1 \& J3) - L2(J2 \& J6) - L3(J4 \& J5) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J11 \& J12) - L7(J13) \}$$

$$G17= \{L1(J11 \& J2) - L2(J3 \& J4) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J1 \& J12) - L7(J13) \}$$

$$G18= \{L1(J1 \& J3) - L2(J4 \& J5) - L3(J6 \& J9) - L4(J7 \& J8) - L5(J10 \& J2) - L6(J11 \& J12) - L7(J13) \}$$

$$G19= \{L1(J2 \& J3) - L2(J4 \& J5) - L3(J6 \& J9) - L4(J7 \& J8) - L5(J10 \& J1) - L6(J11 \& J12) - L7(J13) \}$$

$$G20= \{L1(J1 \& J4) - L2(J2 \& J3) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J11 \& J12) - L7(J13) \}$$

$$G21= \{L1(J1 \& J6) - L2(J2 \& J3) - L3(J4 \& J5) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J11 \& J12) - L7(J13) \}$$

$$G22= \{L1(J11 \& J1) - L2(J3 \& J4) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J2 \& J12) - L7(J13) \}$$

7.4. Case -IV

In this case maximum sub-lots are fourteen. The best group schedules for this case are listed below. The each lot completion time of the each best group schedule and its makespan (C_j) shown in Table 7.

$$G1= \{L1(J1 \& J2) - L2(J13 \& J4) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J9) - L6(J11 \& J12) - L7(J14 \& J3)\}$$

$$G2= \{L1(J1 \& J2) - L2(J3 \& J4) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J10 \& J13) - L6(J11 \& J12) - L7(J14 \& J9)\}$$

$$G3= \{L1(J1 \& J2) - L2(J3 \& J4) - L3(J5 \& J6) - L4(J7 \& J8) - L5(J13 \& J9) - L6(J11 \& J12) - L7(J10 \& J14)\}$$

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- G4= {L1(J1 & J2) - L2(J3 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J11) - L6(J9 & J12) - L7(J13 & J14)}
- G5= {L1(J1 & J2) - L2(J11 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J3 & J12) - L7(J13 & J14)}
- G6= {L1(J1 & J2) - L2(J3 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J11 & J9) - L6(J10 & J12) - L7(J13 & J14)}
- G7= {L1(J1 & J2) - L2(J3 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J13 & J12) - L7(J11 & J14)}
- G8= {L1(J1 & J2) - L2(J3 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J13 & J11) - L7(J12 & J14)}
- G9= {L1(J1 & J2) - L2(J13 & J3) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J11 & J12) - L7(J4 & J14)}
- G10= {L1(J1 & J2) - L2(J3 & J4) - L3(J11 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J5 & J12) - L7(J13 & J14)}
- G11= {L1(J1 & J2) - L2(J3 & J4) - L3(J5 & J11) - L4(J7 & J8) - L5(J10 & J9) - L6(J6 & J12) - L7(J13 & J14)}
- G12= {L1(J1 & J2) - L2(J3 & J11) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J4 & J12) - L7(J13 & J14)}
- G13= {L1(J1 & J2) - L2(J3 & J6) - L3(J5 & J4) - L4(J7 & J8) - L5(J10 & J9) - L6(J11 & J12) - L7(J13 & J14)}
- G14= {L1(J1 & J3) - L2(J2 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J11 & J12) - L7(J13 & J14)}
- G15= {L1(J1 & J3) - L2(J5 & J4) - L3(J2 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J11 & J12) - L7(J13 & J14)}
- G16= {L1(J1 & J3) - L2(J2 & J6) - L3(J5 & J4) - L4(J7 & J8) - L5(J10 & J9) - L6(J11 & J12) - L7(J13 & J14)}
- G17= {L1(J11 & J2) - L2(J3 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J1 & J12) - L7(J13 & J14)}
- G18= {L1(J2 & J3) - L2(J5 & J4) - L3(J9 & J6) - L4(J7 & J8) - L5(J10 & J1) - L6(J11 & J12) - L7(J13 & J14)}
- G19= {L1(J1 & J6) - L2(J2 & J3) - L3(J5 & J4) - L4(J7 & J8) - L5(J10 & J9) - L6(J11 & J12) - L7(J13 & J14)}
- G20= {L1(J1 & J4) - L2(J2 & J3) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J11 & J12) - L7(J13 & J14)}
- G21= {L1(J1 & J11) - L2(J3 & J4) - L3(J5 & J6) - L4(J7 & J8) - L5(J10 & J9) - L6(J2 & J12) - L7(J13 & J14)}

Table 7 Lot completion times and makespan for case-IV best schedules

| G _s | C _{iL1} | C _{iL2} | C _{iL3} | C _{iL4} | C _{iL5} | C _{iL6} | C _{iL7} | C _i |
|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|----------------|
| G ₁ | 1291 | 1429 | 1125 | 1898 | 2123 | 1835 | 1213 | 10914 |
| G ₂ | 1291 | 1371 | 1125 | 1898 | 1719 | 1835 | 1675 | 10914 |
| G ₃ | 1291 | 1371 | 1125 | 1898 | 1685 | 1835 | 1709 | 10914 |
| G ₄ | 1291 | 1371 | 1125 | 1898 | 1744 | 2211 | 1275 | 10915 |
| G ₅ | 1291 | 1457 | 1125 | 1898 | 2123 | 1747 | 1275 | 10916 |
| G ₆ | 1291 | 1371 | 1125 | 1898 | 1713 | 2243 | 1275 | 10916 |
| G ₇ | 1291 | 1371 | 1125 | 1898 | 2123 | 1807 | 1302 | 10917 |
| G ₈ | 1291 | 1371 | 1125 | 1898 | 2123 | 1312 | 1797 | 10917 |
| G ₉ | 1291 | 1223 | 1125 | 1898 | 2123 | 1835 | 1422 | 10917 |
| G ₁₀ | 1291 | 1371 | 1404 | 1898 | 2123 | 1555 | 1275 | 10917 |
| G ₁₁ | 1291 | 1371 | 1060 | 1898 | 2123 | 1899 | 1275 | 10917 |
| G ₁₂ | 1291 | 1251 | 1125 | 1898 | 2123 | 1955 | 1275 | 10918 |
| G ₁₃ | 1291 | 1315 | 1181 | 1898 | 2123 | 1835 | 1275 | 10918 |
| G ₁₄ | 883 | 1779 | 1125 | 1898 | 2123 | 1835 | 1275 | 10918 |
| G ₁₅ | 883 | 1181 | 1723 | 1898 | 2123 | 1835 | 1275 | 10918 |
| G ₁₆ | 883 | 1723 | 1181 | 1898 | 2123 | 1835 | 1275 | 10918 |
| G ₁₇ | 1659 | 1371 | 1125 | 1898 | 2123 | 1467 | 1275 | 10918 |
| G ₁₈ | 1571 | 1181 | 1780 | 1898 | 1379 | 1835 | 1275 | 10919 |
| G ₁₉ | 1037 | 1571 | 1181 | 1898 | 2123 | 1835 | 1275 | 10920 |
| G ₂₀ | 1093 | 1571 | 1125 | 1898 | 2123 | 1835 | 1275 | 10920 |
| G ₂₁ | 975 | 1371 | 1125 | 1898 | 2123 | 2155 | 1275 | 10922 |

7.5. Case -V

In this case maximum sub-lots are ten. The best group schedules for this case are listed below. The each lot completion time of the each best group schedule and its makespan (C_j) shown in Table 8.

$$G_1 = \{L_1(J_1 \& J_2) - L_2(J_{13} \& J_4) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_{11} \& J_{12}) - L_7(J_{14} \& J_4) - L_8(J_{15})\}$$

$$G_2 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_9 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_5) - L_6(J_{11} \& J_{12}) - L_7(J_{14} \& J_{13}) - L_8(J_{15})\}$$

$$G_3 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_{13} \& J_{12}) - L_7(J_{14} \& J_{11}) - L_8(J_{15})\}$$

$$G_4 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_{11} \& J_{13}) - L_7(J_{14} \& J_{12}) - L_8(J_{15})\}$$

$$G_5 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{15} \& J_9) - L_6(J_{11} \& J_{12}) - L_7(J_{14} \& J_{13}) - L_8(J_{10})\}$$

$$G_6 = \{L_1(J_1 \& J_2) - L_2(J_3 \& J_4) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_{11} \& J_{15}) - L_7(J_{14} \& J_{13}) - L_8(J_{12})\}$$

$$G_7 = \{L_1(J_1 \& J_3) - L_2(J_2 \& J_4) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_{11} \& J_{12}) - L_7(J_{14} \& J_{13}) - L_8(J_{15})\}$$

$$G_8 = \{L_1(J_1 \& J_4) - L_2(J_3 \& J_2) - L_3(J_5 \& J_6) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_{11} \& J_{12}) - L_7(J_{14} \& J_{13}) - L_8(J_{15})\}$$

$$G_9 = \{L_1(J_1 \& J_6) - L_2(J_3 \& J_2) - L_3(J_5 \& J_4) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_{11} \& J_{12}) - L_7(J_{14} \& J_{13}) - L_8(J_{15})\}$$

$$G_{10} = \{L_1(J_2 \& J_6) - L_2(J_3 \& J_1) - L_3(J_5 \& J_4) - L_4(J_7 \& J_8) - L_5(J_{10} \& J_9) - L_6(J_{11} \& J_{12}) - L_7(J_{14} \& J_{13}) - L_8(J_{15})\}$$

- G11= {L1(J3 & J6) - L2(J1 & J2) – L3(J5 & J4) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12) – L7(J14 & J13) – L8(J15)}
- G12= {L1(J4 & J5) - L2(J3 & J1) – L3(J2 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12) – L7(J14 & J13) – L8(J15)}
- G13= {L1(J1 & J2) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12) – L7(J14 & J15) – L8(J13)}
- G14= {L1(J1 & J2) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12) – L7(J15 & J13) – L8(J14)}
- G15= {L1(J1 & J2) - L2(J5 & J4) – L3(J9 & J6) – L4(J7 & J8) – L5(J10 & J3) – L6(J11 & J12) – L7(J14 & J13) – L8(J15)}
- G16= {L1(J1 & J2) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J15) – L6(J11 & J12) – L7(J14 & J13) – L8(J9)}
- G17= {L1(J1 & J2) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J15 & J12) – L7(J14 & J13) – L8(J11)}
- G18= {L1(J1 & J2) - L2(J13 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J9) – L6(J11 & J12) – L7(J14 & J3) – L8(J15)}
- G19= {L1(J1 & J2) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J10 & J13) – L6(J11 & J12) – L7(J14 & J9) – L8(J15)}
- G20= {L1(J1 & J2) - L2(J3 & J4) – L3(J5 & J6) – L4(J7 & J8) – L5(J13 & J9) – L6(J11 & J12) – L7(J14 & J10) – L8(J15)}

Table 8 Lot completion times and makespan for case V - best schedules

| Gs | C _{iL1} | C _{iL2} | C _{iL3} | C _{iL4} | C _{iL5} | C _{iL6} | C _{iL7} | C _{iL8} | C _i |
|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|----------------|
| G ₁ | 1291 | 1223 | 1125 | 1898 | 2123 | 1835 | 1422 | 943 | 11860 |
| G ₂ | 1291 | 1371 | 1780 | 1898 | 1467 | 1835 | 1275 | 943 | 11860 |
| G ₃ | 1291 | 1371 | 1125 | 1898 | 2123 | 1807 | 1302 | 943 | 11860 |
| G ₄ | 1291 | 1371 | 1125 | 1898 | 2123 | 1312 | 1797 | 943 | 11860 |
| G ₅ | 1291 | 1371 | 1125 | 1898 | 1987 | 1835 | 1275 | 1079 | 11861 |
| G ₆ | 1291 | 1371 | 1125 | 1898 | 2123 | 1611 | 1275 | 1167 | 11861 |
| G ₇ | 883 | 1779 | 1125 | 1898 | 2123 | 1835 | 1275 | 943 | 11861 |
| G ₈ | 1093 | 1571 | 1125 | 1898 | 2123 | 1835 | 1275 | 943 | 11863 |
| G ₉ | 1037 | 1571 | 1181 | 1898 | 2123 | 1835 | 1275 | 943 | 11863 |
| G ₁₀ | 1723 | 883 | 1181 | 1898 | 2123 | 1835 | 1275 | 943 | 11861 |
| G ₁₁ | 1315 | 1291 | 1181 | 1898 | 2123 | 1835 | 1275 | 943 | 11861 |
| G ₁₂ | 1181 | 883 | 1723 | 1898 | 2123 | 1835 | 1275 | 943 | 11861 |
| G ₁₃ | 1291 | 1371 | 1125 | 1898 | 2123 | 1835 | 1573 | 645 | 11861 |
| G ₁₄ | 1291 | 1371 | 1125 | 1898 | 2123 | 1835 | 1583 | 635 | 11861 |
| G ₁₅ | 1291 | 1181 | 1780 | 1898 | 1659 | 1835 | 1275 | 943 | 11862 |
| G ₁₆ | 1291 | 1371 | 1125 | 1898 | 2019 | 1835 | 1275 | 1048 | 11862 |
| G ₁₇ | 1291 | 1371 | 1125 | 1898 | 2123 | 2107 | 1275 | 672 | 11862 |
| G ₁₈ | 1291 | 1429 | 1125 | 1898 | 2123 | 1835 | 1213 | 943 | 11857 |
| G ₁₉ | 1291 | 1371 | 1125 | 1898 | 1719 | 1835 | 1675 | 943 | 11857 |
| G ₂₀ | 1291 | 1371 | 1125 | 1898 | 1685 | 1835 | 1709 | 943 | 11857 |

8. CONCLUSION

The two Hybrid Flowshop batch scheduling with proposed new heuristics for Level II and for Level I FIFO priority is discussed detailed in this paper. The personalized mathematical model presented for solving this problem some other methods in future. The developed heuristics not only outperforms but also suggested some additional schedules which permits

additional flexibility in execution in real time application with other than the stated constraint. The heuristic solution validated through simulation with case study problem. The queue status examined at critical machine and optimized.

REFERENCES

- [1] Saravanan R. and Dr, Raju R., "Sequencing and scheduling of non-uniform flow pattern in parallel hybrid flow shop", *International Journal of Advanced Manufacturing Technology*, Springer, 49 (1-4), 213-225. (2010).
- [2] Jaehwan Yang, 'A two-stage hybrid flow shop with dedicated machines at the first stage', *Computers & Operations Research*, Vol. 40 (12), pp. 2836-2843. (2013)
- [3] Jianming Dong, Yiwei Jiang, An Zhang, Jueliang Hu and Hui Luo, 'An approximation algorithm for proportionate scheduling in the two-stage hybrid flow shop', *Information Processing Letters*, Vol. 115, Issue 4, pp. 475-480. (2015)
- [4] QiWei Erfang Shan and Liying Kang , 'A FPTAS for a two-stage hybrid flow shop problem and optimal algorithms for identical jobs', *Theoretical Computer Science*, Volume 524, 6 March 2014, Pages 78-89.(2014).
- [5] Chin-Chia Wu, Jia-Yang Chen, Win-Chin Lin, Kunjung Lai and Pay-Wen Yu, 'A two-stage three-machine assembly flow shop scheduling with learning consideration to minimize the flowtime by six hybrids of particle swarm optimization', *Swarm and Evolutionary Computation*, Vol. 41, pp.97-110. (2018).
- [6] Biao Zhang, Quan-ke Pan, Liang Gao, Xin-li Zhang, Hong-yan Sang, Jun-qing Li, An effective modified migrating birds optimization for hybrid flowshop scheduling problem with lot streaming, *Applied Soft Computing*, Vol.52, 14-27 (2017).
- [7] Omid Shahvari and Rasaratnam Logendran, 'A comparison of two stage-based hybrid algorithms for a batch scheduling problem in hybrid flow shop with learning effect', *International Journal of Production Economics*, Vol. 195, pp. 227-248 (2018).
- [8] P Vivek, R Saravanan, M Chandrasekaran, R Pugazhenthhi, "Heuristic for Critical Machine Based a Lot Streaming for Two-Stage Hybrid Production Environment" *IOP Conference Series: Materials Science and Engineering*, Vol.183 (1), 12-30 (2017).
- [9] R.Pugazhenthhi and M. AnthonyXavior, "A Survey on Occurrence of Critical Machines in a Manufacturing Environment", *Procedia Engineering*, Vol. 97, 105-114 (2014).
- [10] Vivek, P., R. Saravanan, M. Chandrasekaran, and R. Pugazhenthhi. "Critical Machine Based Scheduling-A Review." In *IOP Conference Series: Materials Science and Engineering*, vol. 183, no. 1, p.1-11. (2017).
- [11] P Vivek, R Saravanan, M Chandrasekaran, R Pugazhenthhi, "Heuristic for Critical Machine Based a Lot Streaming for Two-Stage Hybrid Production Environment" *IOP Conference Series: Materials Science and Engineering*, 183 (1), 12-30 (2017).
- [12] Dr. Sridhar K and Prakash T. Lazarus, Experimental Design of Constraint Satisfaction Adaptive Neural Network for Generalized Job-Shop Scheduling, *International Journal of Industrial Engineering Research and Development (IJIERD)*, Volume 5, Issue 3, May - June (2014), pp. 24-30.
- [13] Dr. R. Dillibabu and Suresh. K, Development Of Multi-Agent Systems Based Intelligent Fleet Maintenance Scheduling, *International Journal of Industrial Engineering Research and Development (IJIERD)*, Volume 1 Number 1, May - June (2010), pp. 115-126.
- [14] Hymavathi Madivada and C.S.P. Rao, A Review On Non Traditional Algorithms For Job Shop Scheduling, *International Journal of Production Technology and Management (IJPTM)*, Volume 3, Issue 1, January-December (2012), pp. 61-77.