CALCULATING ROTATION ANGLES OF THE OPERATOR'S ARMS BASED ON GENERALIZED COORDINATES OF THE MASTER DEVICE WITH FOLLOWING ANTHROPOMORPHIC MANIPULATOR IN REAL TIME

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ABSTRACT

The purpose of the manuscript is improving the accuracy of motion capture at following control of an anthropomorphic manipulator in real time using a master device, embodied as a lever mechanism with a kinematic scheme similar to a human arm. The generalized coordinates of the master device are used to calculate the Cartesian axials of its nodes using the Denavite-Hartenberg representation. Then, the node coordinates of the operator's arm are determined based on the coordinates of the master device nodes and the existing kinematic links in the Cartesian space. A geometric solution of the inverse kinematics problem is developed to calculate the rotation angles of the operator's arm based on the Cartesian axials of its nodes. The developed method is based on the analytical solution of the problem of calculating the rotation angles of the operator's arm based on the generalized coordinates of the following control master device, which provides low computational complexity. The numerical experiment showed a decrease in the average error in following the operator's motions from 20.7° up to 2.9° a total of all rotating freedom, upon using the developed calculation method. The value of the obtained results consists in the possibility of calculating the rotation angles of the operator's arm based on the generalized coordinates of the following control master device in real time. This method will significantly improve the accuracy of the following control of an anthropomorphic manipulator using master devices embodied as a lever system with links parallel to the operator's arm.

Key words: inverse kinematic problem, geometric approach, anthropomorphic manipulator, three-link manipulator, seven rotating freedoms, the Denavite-Hartenberg representation.

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1. INTRODUCTION

The system of following control of the anthropomorphic manipulator includes: 1) the master device, which is a means for capturing the operator's movement; 2) a computing device that generates control rules for an executive manipulator; 3) the actuator in the form of an anthropomorphic manipulator that is the object of control.

As an analogue, the authors chose FCMD-3 (following control master device) for anthropomorphic robotic systems of AR-600, FEDOR, produced by Npo Androidnaya Tekhnika JSC [1]. The principle for implementing the capture of the operator's arm movements, realized by this device, consists in recording the rotation angles of the turning pairs of the master device. The parallel arrangement of the FCMD-3 lever system relatively to the operator's arm excludes the coincidence of the centers of its kinematic pairs and operator's joints. In this regard, the recorded rotation angles of FCMD-3 links do not coincide with the rotation angles in the operator's joints. As a consequence, the movements of the anthropomorphic manipulator links are not completely identical to those set by the operator. The clamp positions of the manipulator and its links are different from those set by the operator. Adjustment of the capture position is carried out by the operator if visual observation of the anthropomorphic manipulator actions is possible. This peculiarity of FCMD-3 limits the use of following control systems based on it. In some cases, when precise control of the manipulator links position is required, it excludes the possibility of its application. The solution of this problem is possible due to the exact calculation of the rotation angles of the operator's arm. The calculation can be performed based on the readings of FCMD encoders and the analytical solution of the inverse kinematics problem with respect to the operator's arm.

The development basis is the method of determining the approximate Cartesian axials of the operator's arm nodes from the known generalized coordinates of the considered master device [2]. In the process of applying the method [2] to the following control of the anthropomorphic manipulator, the following problems arise. First, the control of the anthropomorphic manipulator is implemented not in Cartesian axials, but in the generalized coordinates representing rotation angles of the turning pairs for the given case. Therefore, mathematical approaches are required to transform the Cartesian axials of the operator's arm nodes into its generalized coordinates used to form the motion laws of the anthropomorphic manipulator. Second, the output data of the method from [2] is not sufficient for unambiguous determination of the generalized coordinates of the operator's arm. Third, increasing the following accuracy in Cartesian axials does not necessarily lead to an increase in the following accuracy in the generalized coordinates.

The goal of the research is to improve the motion capture accuracy in the following control of an anthropomorphic manipulator in real time using a master device embodied as a lever mechanism having a kinematic scheme similar to a human arm.

It is necessary to solve the following tasks to achieve this goal:
1. Adapt the method for calculating the Cartesian axials of the operator's arm [2] by calculating the necessary additional values.
2. Develop a way of transforming the Cartesian axials of the operator's arm into its generalized coordinates.
3. Develop a method for calculating the generalized coordinates of the operator's arm based on the generalized coordinates of the master device.
2. LITERATURE REVIEW

The methods of solving the inverse kinematics problem by the underlying methods can be divided into two large groups - numerical and analytical.

Numerical optimization methods for solving the inverse kinematics problem of three-link manipulators with 7 rotating freedoms are provided in manuscripts by [3-11]. The manuscripts by [3, 9, 12] use numerical simulation methods to solve the problem. The manuscripts by [4, 5, 7, 8] use the Jacobi matrices as the basis for simulating the manipulator's kinematics, and the manuscripts by [6, 11, 12, 14] use the Denavite-Hartenberg representation for the same purpose. The manuscripts by [4, 14] apply optimization methods based on gradient methods, particle swarms [5, 6, 13], and genetic algorithms [9-13]. Neural networks in solving the inverse kinematics problem were reflected in [6, 12, 13]. Numerical methods for solving the inverse kinematics problem allowed for obtaining solutions optimal with the given criteria with an acceptable accuracy degree. The main disadvantage of numerical methods is the high computational complexity of the algorithms for their implementation and, as a consequence, high requirements for computing resources for operation in real time.

The application of analytical methods for solving the inverse kinematics problem is presented in publications by [15-23]. The Denavite-Hartenberg representation is used in the manuscripts by [15, 18, 21, 22], and the Jacobi matrices are used in the manuscripts by [16, 17]. The manuscripts by [20, 23] use intuitive estimation and maneuverability estimation to solve the inverse kinematics problems, respectively. Geometric methods for solving the inverse kinematics problem are described in the manuscripts by [20-23]. The presented methods can work in real time, and their effectiveness is demonstrated in numerical examples.

The advantage of using analytical methods for solving the inverse kinematics problem lays in their low computational complexity, which allows for using them in systems operating in real time. However, as a rule, analytical methods are applicable only to a small number of individual problems.

Since the developed method is oriented to the following control in real time, a restriction on the acceptable computational complexity of the chosen method for solving the inverse kinematics problem is imposed. Therefore, a geometric approach was chosen based on the Denavite-Hartenberg representation to transform the Cartesian axials of the operator's arm into the generalized coordinates, because this approach has low computational complexity, and the Denavite-Hartenberg representation has good interpretability. On the other hand, the specificity of the following control problem makes allows for eliminating such drawbacks of the geometric approach as the existence of acceptable solutions set of the inverse kinematics problem.

3. MATERIALS AND METHODS

3.1. Calculation of the Cartesian axials of the operator's arm nodes based on the generalized coordinates of the master device

The authors consider the model of an anthropomorphic manipulator as a kinematic model of the operator's arm, which has the following imposed restrictions. The lengths of the manipulator model links shall be equal to the operator's arm segment lengths, the kinematic scheme shall be similar to a human arm, and the nodes shall be located in the same points of the volume space as the operator's arm nodes. Nodes shall mean the centers of the joints and the middle of the operator's arm. Figure 1 shows the kinematic scheme of the operator, where
$B_1 - B_3$ is the shoulder, elbow, and radio-carpal nodes; $B_4$ is the node of the hand centre; $A_1 - A_7$ is the turning pairs. $A_1$ is the node of the hand centre; $A_2, A_3, A_4, A_5, A_6, A_7$ are the corresponding nodes of the master device, which has a similar kinematic scheme.

Known values for the operator's hand are the lengths of the shoulder, elbow and carpal parts $l_{B_1-B_2}, l_{B_2-B_3}, l_{B_3-B_4}$, respectively. The length data of $l_{B_i-B_j}$ type correspond to the distance between the nodes $B_i$ and $B_j$ of the kinematic model. The authors use the Denavit-Hartenberg representation as a mathematical model of an anthropomorphic manipulator, which is a kinematic model of the operator's arm. The location and orientation of the coordinate systems associated with the links are shown in Figure 2. In accordance with the adopted kinematic model of the operator's arm, its generalized coordinates are the rotation angles in turning pairs. Therefore, the position of the operator's arm is uniquely determined by the angular values vector: $\theta = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7)^T$.

**Figure 1** Shows operator's arm geometry  **Figure 2** Shows links related to the coordinates systems

Figure 3 shows the closed circuit kinematics formed by FCMD and the operator's arm. The letters $B_1, B_2, B_3, B_4$ designate the shoulder, elbow, radio-carpal and carpal nodes of the operator, the letters $C_1, C_2, C_3, C_4$ designate the corresponding nodes of the master device, which has a similar kinematic scheme.

**Figure 3** Shows kinematic diagram of the concatenated operator's arm and the master device; $O$ is the attachment point of the master device to the operator's body

The operator's arm and the master device are concatenated in several places by rigid segments. Rigid coupling $B_1OC_1$ prevents mutual movement of the operator's shoulder joint $B_1$ the shoulder joint of the master device $C_1$. Segment $B_3B_4C_4C_3$ unites the carpal links of the operator and master device for better control over the exoskeleton movement. There is also a
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Flexible coupling, represented in Figure 3 as a kinematic chain \( B_5C_5 \). This chain holds the exoskeleton links in the parallel parts of the operator’s arm position. Otherwise, the arm position shown in Figure 3 may be found in a situation when the elbow joint is pointing downwards, and the elbow joint of the exoskeleton is pointing upwards.

Known data for the master device and the operator’s arm are the following values:

- Coordinates of the point \( B_1 = (0;0;0) \), taken as the reference point;
- Vector coordinates of rigid coupling \( B_1C_1 = (x_{B1C1}; y_{B1C1}; z_{B1C1}) \);
- Lengths of the operator’s arm parts \( l_{B1-B2}, l_{B2-B3}, l_{B3-B4} \);
- Lengths of the master device segments \( l_{C1-C2}, l_{C2-C3}, l_{C3-C4} \);
- Lengths of couplings \( B_2C_2, B_3C_3 \) and \( B_4C_4 = l_{B2-C2}, l_{B3-C3}, l_{B4-C4} \).

Vector of the measured values of the generalized coordinates of the master device: \( \theta' = (\theta'_1, \theta'_2, \theta'_3, \theta'_4, \theta'_5, \theta'_6, \theta'_7)^T \).

The authors use the Denavite-Hartenberg representation to construct the mathematical model of the master device. Master device kinematics is shown in Figure 4. The position and orientation of the coordinate systems associated with the links are shown in Figure 5.

\[ T_i = T_0 \ 0T_i, \ i > 0, \ T_0 = T_x(-90^\circ)T_x(-90^\circ), \]  

Figure 4 shows master device kinematics  
Figure 5 shows location and orientation of the coordinate systems linked to the segments

Since the kinematics of the operator’s arm and the master device are similar, and both schemes are treated with the Denavite-Hartenberg formalism, the following are the notations for quantities related to the operator’s arm, implying similar values for the master device. The stroke symbol ”” is applied to distinguish the designations related to the master device.

Let us assume the following designations:

\( \ 0T_i \) is a homogeneous transformation matrix from the i-th coordinate system to \( i, \ i < j \), compiled in accordance with the Denavite-Hartenberg representation;

\( T_i \) is the transformation matrix into the global coordinate system from the i-th coordinate system, which can be found by the following formula:

\[ T_i = T_0 \ 0T_i, \ i > 0, \ T_0 = T_x(-90^\circ)T_x(-90^\circ), \]
The basic formulas for calculating the coordinates of the operator's arm nodes:

\[
T_i' = T_0' 
\begin{pmatrix}
T_{B1-C1} & T_0 \cdot C(\theta) \cdot T_x(\theta) \cdot T_y(\theta) \\
1 & 0 & 0 & B_1 \cdot C_{1x} \\
0 & 1 & 0 & B_1 \cdot C_{1y} \\
0 & 0 & 1 & B_1 \cdot C_{1z} \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

where:

- \( K_i \) - uniform radius-vector of the point \( K_i \) in \( i \)-th coordinate system;
- \( K_j \) - uniform radius-vector of the point \( K_j \) in the global coordinate system;
- \( K_{jx}, K_{jy}, K_{jz} \) are projections of the radius-vector \( K_j \) at the axis \( x, y, z \) of \( i \)-th coordinate system;
- \( K_{jx}, K_{jy}, K_{jz} \) are projections of the radius-vector \( K_j \) at the axis \( x, y, z \) of the global coordinate system.

The basic formulas for calculating the coordinates of the operator's arm nodes:

\[
C_2 = T_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},
B_3 = T_6 \begin{pmatrix} 0 \\ -l_{B3-C3} \\ 1 \end{pmatrix},
B_4 = T_7 \begin{pmatrix} 0 \\ -l_{B4-C4} \\ 1 \end{pmatrix},
\]

\[
l_{K2-B2} = \frac{2}{l_{B1-B3}} \sqrt{p(p - l_{B1-B3})},
\]

\[
p = \frac{t_{B1-B3} + t_{B2-B3} + t_{B1-B2}}{2}.
\]

The basic formulas for calculating the coordinates of the operator's arm nodes:

\[
\lambda = \frac{t_{B1-B2}}{l_{B2-B3}},
\]

\[
K_2 = \begin{pmatrix} x_{B1} + \lambda x_{B3} \\ y_{B1} + \lambda y_{B3} \\ z_{B1} + \lambda z_{B3} \end{pmatrix},
\]

\[
B_1B_3 = \begin{pmatrix} x_{B1B3} \\ y_{B1B3} \\ z_{B1B3} \end{pmatrix},
B_1B_2 = \begin{pmatrix} x_{B2} \\ y_{B2} \\ z_{B2} \end{pmatrix},
\]

\[
C_2 = \begin{pmatrix} x_{C2} \\ y_{C2} \\ z_{C2} \end{pmatrix},
\]

\[
B_2a = \begin{pmatrix} x_{B2a} \\ y_{B2a} \\ z_{B2a} \end{pmatrix},
B_2b = \begin{pmatrix} x_{B2b} \\ y_{B2b} \\ z_{B2b} \end{pmatrix},
\]

\[
a = \frac{x_{B1B3}}{B_1B_3},
b = \frac{x_{B1B3}}{B_1B_3},
c = \frac{x_{B1B3}}{B_1B_3},
r = \frac{x_{B1B3}}{B_1B_3},
m = \frac{l_{K2-B2}^2 - 2c(K_{i2} - C_{i2}) - C_{i2}^2 - C_{i2}^2 - K_{i2}^2 + K_{i2}^2 + K_{i2}^2}{2[(K_{i2} - C_{i2})]}.
\]

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\[ q = 2n(l - C_{2i}) + 2r(m - C_{2j}) - 2C_{2k}, \]
\[ s = (l - C_{2i})^2 + (m - C_{2j})^2 + C_{2k}^2 - l_{22}^2 - C_{2}, \]
\[ t = q^2 - 4 \cdot p \cdot s, \]
\[ B_{2a_k} = \frac{-1 - \sqrt{t}}{2p}, \quad B_{2a_j} = r \cdot B_{2a_k} + m, \quad B_{2a_i} = a \cdot B_{2a_j} + b \cdot B_{2a_k} + c, \]
\[ B_{2b_k} = \frac{-1 + \sqrt{t}}{2p}, \quad B_{2b_j} = r \cdot B_{2b_k} + m, \quad B_{2b_i} = a \cdot B_{2b_j} + b \cdot B_{2b_k} + c, \]

Indices \( i, j, k \) in the given expressions are selected as follows. The index \( i \) has the number of the first non-zero component of the vector \( B_2 B_3 \), (the numbering starts at 0), the remaining indices are equal to the number of the remaining components, and the order can be arbitrary.

\( B_{2a} \) and \( B_{2b} \) is the radius vectors drawn into possible positions of the elbow joint of the operator's arm. The choice of one of the values is based on the selection made at the previous iteration of executing the method:

\[ B_{2}^{(n+1)} = \begin{cases} B_{2a}^{(n+1)}, & \text{если } B_{2a}^{(n)} B_{2a}^{(n+1)} \leq B_{2b}^{(n)} B_{2b}^{(n+1)} \\ B_{2b}^{(n+1)}, & \text{если } B_{2b}^{(n)} B_{2a}^{(n+1)} > B_{2b}^{(n)} B_{2b}^{(n+1)} \end{cases} \]  

(13)

The coordinates of the normal vector to the operator's palm emerging from its inner side are needed to transform the Cartesian axials of the operator's arm into its generalized coordinates, in addition to the Cartesian axials of the operator's hand \( B_1 - B_4 \) nodes. Vector \( N = -B_3 C_3 \) can be taken as such vector. Radius-vector \( C_3 \) of the beginning of this vector can be calculated by the formula:

\[ C_3 = T_6^0 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \]  

(14)

Formulas (2) – (14) allow for calculating the Cartesian axials of all the points needed to calculate the generalized coordinates of the operator's arm in the next Subsection 3.2. The shown analytical expressions have fixed and low computational complexity, which allows for using them in real time. The calculation accuracy for the coordinates of the points \( B_3, B_4 \) and \( C_3 \) is determined by the input data error. The error in calculating the coordinates of the point \( B_2 \) is methodological, since the general idea of calculation is based on empirical observation.

3.2. Calculation of the rotation angles of the operator's arm based on the Cartesian axials of its nodes

The purpose of the calculation is to determine the generalized coordinates of the operator's arm \( \Theta = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7\} \), for which the node points are at the points of space \( B_1 - B_4 \) found earlier, and the normal vector to the plane of the palm is co-directed with the vector \( N \).

The calculation process shall be done as follows. Starting from the shoulder, let us successively calculate the generalized coordinates of the operator's arm. In this case, after each calculated coordinate, let us move to a new coordinate system, in which the process of searching for the next generalized coordinate is facilitated, etc.

Let us denote the generalized coordinates vector in the arm position, shown in Fig. 1, as \( \Theta = \{\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7\} \).
The operator’s arm has some limitations of rotating freedom, which can be approximated as follows. The width of the variation range of each generalized coordinate is about 180°. And in the position shown in Figure 1, the values of each coordinate lay in the middle of the acceptable values range. The following restrictions are put on the generalized coordinates of the operator’s arm:

\[ \theta_i - 90^\circ \leq \theta_i \leq \theta_i + 90^\circ. \]  

(15)

Since the considered range of changes is less than \( \pi \) radian, the specified range gets only one of several solutions, which can be obtained by a geometric method for solving the inverse kinematics problem, developed below. Therefore, the adopted restrictions allow for levelling out the drawbacks of the geometric approach, as noted in Section 2.

Further the function \( \text{atan}2(x, y) \) is used - the arc tangent of two arguments, which allows for considering the quadrant of the angular argument:

\[
\text{atan}2(x, y) = \begin{cases} 
\arctan(y/x), & \text{если } x \geq 0, y \geq 0 \\
\arctan(y/x), & \text{если } x > 0, y < 0 \\
\arctan(y/x) - \pi, & \text{если } x < 0, y < 0 \\
\arctan(y/x) + \pi, & \text{если } x < 0, y > 0
\end{cases}
\]  

(16)

As can be seen from Figure 2, the position of the node \( B_2 \) is determined by rotations around the axes \( z_0 \) and \( z_1 \), that is, the rotation angles \( \theta_1 \) and \( \theta_2 \). Rotation around the axis \( z_2 \) and the remaining axes only affects the coordinates of the points farther from the shoulder joint than \( B_2 \). The calculation of the coordinates \( \theta_1 \) and \( \theta_2 \) is similar to the calculation of the spherical coordinates of the vector deferred from the origin of the Cartesian axials.

Let us find the coordinates of point \( B_2 \) in the 0nd coordinate system:

\[ B_0^2 = (T_0)^{-1}B_2. \]  

(17)

The angle \( \theta_1 \) is the angle between the axis \( x_0 \) and the projection of the axis \( x_1 \) on the plane \( x_0Oy_0 \). This projection is always aligned with the projection of link 2 on the same plane. The links 2 and 3 lay on one straight line, so the angle \( \theta_1 \) can be found as the angle between the axis \( x_0 \) and the projection of the point \( B_2 \) on the plane \( x_0Oy_0 \):

\[ \theta_1^* = \text{atan}2(B_2^0x, B_2^0y), \theta_1 = \begin{cases} 
\theta_1^*, & \text{если } \theta_1^* > 90^\circ \\
\theta_1^* + 2\pi, & \text{если } \theta_1^* < 90^\circ.
\end{cases} \]  

(18)

The intermediate value \( \theta_1^* \) and formulas (4) are introduced to bring the obtained value to the range of change \( \theta_1 \).

Knowing the rotation angle \( \theta_1 \), it is possible to shift to the coordinate system associated with the first link:

\[ B_1^2 = (T_1)^{-1}B_2. \]  

(19)

The angle \( \theta_4 \) is the angle between the vectors \( x_1 \) and \( x_2 \), counted along the axis \( z_1 \). The axis \( x_2 \) is connected with the second link and is perpendicular to it. In turn, the link in the initial position is aligned with the axis \( x_1 \). Therefore, the change in the rotation angle of the axis \( x_2 \) in comparison with the initial position will be equal to the azimuth angle of the point \( B_3^1 \). Considering the initial position of the arm, the second generalized coordinate can be found as:

\[ \theta_2 = \theta_2 + \text{atan}2(B_2^1x, B_2^1y). \]  

(20)

Let us consider the position of the radio-carpal joint point \( B_3 \). With the calculated generalized coordinates of \( \theta_1 \) and \( \theta_2 \), its position is uniquely determined by the angle \( \theta_3 \).
corresponding to the rotation along the shoulder link, and $\theta_4$ corresponding to the bend in the elbow.

The angle $\theta_3$ is equal to the angle between the axes $x_2$ and $x_3$, counted along the axis $z_2$. At the same time, the axis $x_3$ is opposite to the forearm vector $B_2B_3$. Thus, the angle $\theta_3$ can be found as the azimuth angle of the vector $B_2B_3$ in the second coordinate system:

$$B_2^2B_3^2 = (T_2)^{-1}B_2 - (T_2)^{-1}B_3, \quad \theta_3 = atan2(B_2^2B_2^2x, B_2^3B_2^2y).$$  \hspace{1cm} (21)

Similarly to the shoulder link, let us move to the third coordinate system and find the fourth generalized coordinate as the forearm link rotation considering the value in the initial position:

$$B_3^3 = (T_3)^{-1}B_3, \quad \theta_4 = \theta_4 + atan2(-B_3^2x, B_3^2y).$$  \hspace{1cm} (22)

Let us consider the end of the bound vector of the normal to the palm in the fourth coordinate system:

$$N^4 = -(T_4)^{-1}C_3.$$  \hspace{1cm} (23)

When rotating along the axis $z_4$, the rotation angle $\theta_5$ corresponds to the rotation of the normal vector projection to the palm on the plane $x_4O_4y_4$. Since the projection of a given normal is always co-directed with the projection onto the same axis $x_5$ plane, the coordinate $\theta_5$ is equal to the azimuthal angle of the given normal in the fourth coordinate system:

$$\theta_5 = atan2(N_4^5, N_4^4).$$  \hspace{1cm} (24)

Let us shift to the fifth coordinate system:

$$N^5 = -(T_5)^{-1}C_3.$$  \hspace{1cm} (25)

Since the vector $N$ is perpendicular to the axis $x_5$, and both vectors lay in a plane perpendicular to $z_6$, the change in their rotation angles is equal to the initial position. Therefore, $\theta_6$ can be calculated similarly to $\theta_2$:

$$\theta_6 = \theta_6 + atan2(N_5^6, N_5^5).$$  \hspace{1cm} (26)

The rotation angle $\theta_7$ is equal to the angle to which the axis $x_6$ shall be rotated along the axis $z_6$ to let it coincide with the axis $x_7$. Since the axis $x_7$ is opposite to the radius vector $B_4^6$, the coordinate $\theta_7$ can be found as the azimuth angle of the corresponding oppositely directed radius vector:

$$B_4^6 = (T_6)^{-1}B_4, \quad \theta_7 = atan2(-B_4^6x, -B_4^6y).$$  \hspace{1cm} (27)

The derived formulas (17) - (27) allow for calculating the generalized coordinates value of the operator's arm from the nodes coordinates, which expression for calculation was obtained in Section 3.1. The proposed solution, based on the geometric approach to solve the inverse kinematics problem, has low computational complexity, which allows for calculations in real time. The imposed restrictions (15), which are acceptable for the considered problem, allow for eliminating the problem inherent in the geometric approach with the presence of several solutions of the inverse kinematics problem. Thus, the second research task can be considered as solved.

### 3.3. The calculating method for the rotation angles of the operator's arm based on the generalized coordinates of the master device

This calculation method for the generalized coordinates of the operator's arm, which are the rotation angles, generalizes the mathematical calculations of the previous sections.
The input data for the offered method are:

- coordinates of the point \( B_4 = (0; 0; 0) \), taken as the reference point;
- vector coordinates of rigid coupling \( B_1C_1 = (B_1C_{1x}; B_1C_{1y}; B_1C_{1z}) \);
- coordinates of the values vector of the generalized coordinates of the master device: \( \theta' = \{ \theta'_1, \theta'_2, \theta'_3, \theta'_4, \theta'_5, \theta'_6, \theta'_7 \} \);
- lengths of the operator's arm parts \( l_{B_1-B_2}, l_{B_2-B_3}, l_{B_3-B_4} \);
- lengths of the master device segments \( l_{C_1-C_2}, l_{C_2-C_3}, l_{C_3-C_4} \);
- lengths of couplings \( B_2C_2, B_3C_3 \) and \( B_4C_4 \).

The output data is the rotation angles of the operator's arm: \( \theta = \{ \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7 \} \).

The developed method can be divided into two integrated stages:

- Calculation of the Cartesian axials of the operator's arm nodes.
- Calculation of the generalized coordinates of the operator's arm according to the Cartesian axles of its nodes.

The essence of the method consists in calculating the necessary points of the operator's arm at the first stage of the Cartesian axials using the method developed in [2] and briefly presented in Subsection 3.1 in the form of the formulas (2) - (14).

The result of the first stage is the calculation of the Cartesian axials of the operator's arm nodes in the global coordinate system - \( B_1, B_2, B_3, B_4 \), and the end of the normal vector to the palm \( N \). The output of the first stage is the input for the second stage.

At the second stage, the Cartesian axials of the operator's arm nodes are transformed into the generalized coordinates using the formulas (19) - (29) developed in Subsection 3.2. The scheme for calculating the rotation angles of the operator's arm based on the generalized coordinates of the master device is shown in Fig. 6.

Figure 6 Shows enlarged scheme of the developed method

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The solutions of the performed tasks used at both stages are analytical, so the resulting computational complexity is low enough that this method can be used in real time. As shown by the computational experiment described in the next subsection, this method can achieve a significant increase in the following accuracy as required by the set goal. Thus, the third task of the study can be considered as solved.

4. RESULTS

In order to verify the effectiveness of the developed method, as well as to validate the formulas of Section 3.2, the authors performed a computational experiment on determining the error in calculating the generalized coordinates of the operator's arm based on the generalized coordinates of the master device. The general concept of simulation is illustrated in Figure 7 and consists in the following. The model consists of a simulation model of the linked operator's arm (OA) and the master device (MD), as well as the computational units implementing the described method. The input of the model received values of generalized coordinates (GC), simulating the real values of the generalized coordinates of the operator's arm. The simulation model of the operator's arm is the leading one and assumes a position corresponding to these coordinates. The simulation model of the master device is set in motion by a simulation model of the operator's arm, and is intended for recording the generalized coordinates of the master device. The measured coordinates of the master device are delivered to the input of the blocks implementing the described method. The calculated values of the generalized coordinates obtained at the output of these blocks are compared with the input values of the generalized coordinates of the operator's arm that are accepted in the simulation as real. The difference between the real and calculated values is the error of the proposed calculation method.

Simulation was carried out in the software package for solving technical computing problems MATLAB v.2016, using the Simscape Multibody add-on. The structure of the developed simulation is shown in Figure 8. The State enumerator block searches the test patterns of the generalized coordinates of the operator's arm to simulate its movement, and sends them to the input of the simulation model block of the concatenated operator's arm and the Kinematic model exoskeleton. The test sample comprises of 2187 vectors of real generalized coordinates $\mathbf{\theta}^* = \{\theta_1^*, \theta_2^*, \theta_3^*, \theta_4^*, \theta_5^*, \theta_6^*, \theta_7^*\}$. The vectors for the test sample are represented as all possible combinations of the following values of generalized coordinates: $\theta_1^* = \{-0.1; 0; 0.1\}$, $\theta_2^* = \{-0.1; 0; 0.1\}$, $\theta_3^* = \{-0.1; 0; 0.1\}$, $\theta_4^* = \{-0.1; 0; 0.1\}$, $\theta_5^* = \{-0.3; 0; 0.3\}$, $\theta_6^* = \{-0.3; 0; 0.3\}$, $\theta_7^* = \{-0.3; 0; 0.3\}$, i.e. 2187 values.
The Kinematic model block implements a simulation model of the linked operator's arm and the master device. The Method block calculates the Cartesian axials of the master device. The Configuration block sets the starting position of the operator's arm and the flexible coupling length $l_{B5-5}$. The Method inverse kinematic and Exoskeleton inverse kinematic blocks transform the Cartesian axials of the operator's arm into generalized coordinates. The Angle Accuracy block calculates the error for the developed method and for the analogue device, and writes the resulting values to a file. The Cartesian Accuracy and Hand inverse kinematic blocks are designed to check the compiled model.

With the presented model, statistical parameters of errors are calculated for different flexible coupling lengths. The obtained results are shown in Table 1. For the value of $l_{B5-5} = 8.0$ cm, not all vectors of the input values have been solved, so this value is not suitable for use.

<table>
<thead>
<tr>
<th>Length of flexible coupling $l_{B5-5}$, cm</th>
<th>Selected analogue error</th>
<th>Selected method error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Deviation</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8,0</td>
<td>20.7°</td>
<td>8.1°</td>
</tr>
<tr>
<td>9.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An increase in the error in the proposed method is observed with increasing coupling length. This is explained by the fact that the distance in the Cartesian axials between the real location of the elbow joint of the operator's arm and the variants of its location calculated according to the formulas (16) increases with the increase in the assumed value of the flexible coupling.

Thus, the optimal value is the flexible coupling length, equal to 9.0 cm, at which the average value of the total error for all generalized coordinates is 2.9°. A similar error for the current methodology used in FCMD-3, according to the simulation, is 20.7°. The developed method is based on the analytical solution of all the necessary problems. Thus, the algorithm
implementing the developed method will have a fixed number of mathematical operations (of $10^4$ order), which allows for using this method in following control systems in real time. According to the simulation, the average total for all generalized coordinates of the developed method error is $2.9^\circ$, while for the analogue device the same error equals to $20.7^\circ$.

5. DISCUSSIONS

The simulation results allow for expecting that the developed method will significantly increase the following motion accuracy with the help of master devices similar to FCMD-3. In addition, the algorithm that implements the method has low computational complexity, which allows for using it following control systems in real time. Despite the low computational complexity, the method has the potential to perform additional optimization for cases where even lower computational costs are required.

The simulation model of the master device was not verified for compliance with FCMD-3; therefore, it is planned to implement the software implementation of the developed method and conduct a full-scale experiment on a real device. The simulation was carried out not in the entire range of admissible values of the generalized coordinates of the operator's arm, due to various kinds of collisions, which appeared in the given simulation model. Therefore, it is necessary either to improve the model, or to estimate the error when applying in a real device.

6. CONCLUSIONS

This manuscript developed a calculating method for the rotation angles of the operator's arm based on the generalized coordinates of the master device in order to increase the following control precision. This method can be divided into two stages - the Cartesian coordinates calculation for the operator's arm nodes based on the generalized coordinates of the master device, and the generalized coordinates calculation for the operator's arm based on the Cartesian coordinates of its nodes. The transformation of the Cartesian coordinates of the operator's arm into its generalized coordinates (formulas (21) - (31)) was developed for the second stage based on the geometric solution to the inverse kinematics problem. The use of the geometric approach allows achieving low computational complexity, and the limitations that are specific to the problem being solved allow eliminating the problem of the acceptable solutions set.

Analytical solutions in both stages motivates low computational complexity of the method (around $10^4$ of operations), which allows using the developed method for following control system operating in real time. The numerical experiment showed a significant increase in the following control accuracy as compared with the analogue in question. The chosen analogue has the average value of the total generalized coordinates as $20.7^\circ$, while the developed method has only $2.9^\circ$. The research goal is achieved, the set tasks are fulfilled.

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REFERENCES


Calculating Rotation Angles of the Operator’s Arms Based on Generalized Coordinates of the Master Device with Following Anthropomorphic Manipulator in Real Time


