International Journal of Mechanical Engineering and Technology (IJMET)
Volume 9, Issue 3, March 2018, pp. 101–111, Article ID: IJMET_09_03_012
Available online at http://www.iaeme.com/IJMET/issues.asp?JType=IJMET&VType=9&IType=3
ISSN Print: 0976-6340 and ISSN Online: 0976-6359

 Scopus Indexed

GENERATING FUNCTIONS FOR HYPERGEOMETRIC POLYNOMIALS OF TWO VARIABLES $R_n(\beta;\gamma;x,y)$ BY TRUESDELL METHOD

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ABSTRACT

In this paper, the Truesdell method is utilized to derive the generating functions for the generalized hypergeometric polynomial set $R_n(\beta;\gamma;x,y)$ *by giving suitable interpretation to the index n. Further, it is interesting to note that these generating functions can be suitably applied to yield numerous applications to various classical orthogonal polynomials of mathematical physics namely the Laguerre, Meixner, Gottlieb and Krawtchouk polynomials. Many of these applications are known but some of them are believed to be new in the theory of special functions.*

Mathematics Subject Classification (2010): *Primary 33C10, secondary 33C45, 33C80*

Key words: Special Functions, Generalized Hypergeometric Polynomials, Generating Functions

Cite this Article: Tadikonda Srinivasulu and V.S. Bhagavan, Generating Functions for Hypergeometric Polynomials of Two Variables $R_n(\beta;\gamma;x,y)$ by Truesdell Method, *International Journal of Mechanical Engineering and Technology* 9(3), 2018, pp. 101–111.
http://www.iaeme.com/IJMET/issues.asp?JType=IJMET&VType=9&IType=3

1. INTRODUCTION

1.1. Definition and Special Cases

Generating functions play an important role in the investigation of various useful properties of the sequences which they generate. They are used with good effect for the determination of the asymptotic behaviour of the generated sequence $\{f_n\}_{n=0}^{\infty}$ as $n \to \infty$. In recent years, the development of advanced computers has made it necessary to study the hypergeometric polynomials with series representations from the numerical point of view. Because of the important role which hypergeometric polynomials play in problems of applied mathematics, the theory of generating functions has been developed various directions and found wide applications in different branches of science and technology.

The generalized hypergeometric polynomials have many applications in different branches of engineering and science (Arfen 1985; Ismail et al. 1997) [1, 5]. A few of them, for example, in mechanical engineering, the solutions of the equations of motion are given using special functions and classical orthogonal polynomials, namely the Laguerre, Meixner, Gottlieb, Krawtchouk polynomials, Bessel functions etc. The non-dimensional frequencies of the beam are given in terms of the cross sectional area and the flexural rigidity at the first end of the beam are obtained using classical orthogonal polynomials (El-Din 2000) [4]. Moreover, in mechanical engineering some modelling problems like constructing a continuous hereditary creep model for the thermoviscoelastic contact of a rough punch and a smooth surface of a rigid half-space can be studied by hypergeometric functions and classical orthogonal polynomials. Generating functions and classical orthogonal polynomials plays a significant role in tribology, that is, in problems of friction, wear, lubrication and contact (Osama et al. 2010) [7].

Many authors (Bride 1971; Dattoli 2000; Rainville 1960; Srivastava et al. 1984; Truesdell 1948) [2, 3, 8, 11, 13] studied the generating functions for the generalized hypergeometric polynomials by various methods. In this paper, the Truesdell's method has been used to derive generating relations for the hypergeometric polynomial set $R_n(\beta;\gamma;x,y),$ by giving interpretation to the index *n*. The Truesdell method has the advantage of getting generating functions independently from ascending and descending differential-difference equations. This method includes the gamma function and it is based upon the Truesdell's functional equations given by

$$\frac{\partial}{\partial z}F(z,\alpha)=F(z,\alpha+1),$$

and

$$\frac{\partial}{\partial z}G(z,\alpha)=G(z,\alpha-1) \quad . \tag{1}$$

These equations are called the *F* and *G*-equations respectively. This yield many valuable results.

For obtaining an ascending generating relation for the set $\{F_{k+n}(x)\}$ (*k* is fixed), Truesdell [12] had used the following ascending differential-difference equation:

$$\frac{d}{dx}F_n(x)=A(x,n)F_n(x)+B(x,n)F_{n+1}(x) \tag{2}$$

where the coefficients $A(x,n)$ and $B(x,n)$ are restricted.

Similarly, for deducing descending generating relation for the set $\{F_{k-n}(x)\}$ (*k* is fixed), the following descending differential-difference equation has been used:

$$\frac{d}{dx}F_n(x)=C(x,n)F_n(x)+D(x,n)F_{n-1}(x), \tag{3}$$

where the coefficients $C(x,n)$ and $D(x,n)$ are restricted.

In this present paper, we have utilized this method and obtained the generating relations for the polynomial set $R_n(\beta;\gamma;x,y)$ with suitable interpretation to the index n and then followed by its applications.

1.2. Special Cases

The following special cases of $R_n(\beta;\gamma;x,y)$ have been obtained:

1. Substituting $\gamma=1+\alpha$, writing $\left(x-\frac{\beta x^2}{y}\right)$ for y and taking $\beta\to\infty$, we have, $\underset{\beta\to\infty}{Lt}\left\{\left(x-\frac{\beta x^2}{y}\right)^{-n}R_n\left(\beta;1+\alpha;\,x,\,x-\frac{\beta x^2}{y}\right)\right\}=\frac{n!}{(1+\alpha)_n}y^{-n}L_n^{(\alpha)}(x,y)$, where $L_n^{(\alpha)}(x,y)$ are the two variable Laguerre polynomials. (Dattoli 2000) [3]

2. Substituting $\beta=-z, y=(1-\rho)^{-1}$, we have,
$R_n(-z;\gamma;1,(1-\rho)^{-1})=(1-\rho)^{-n}M_n(z;\gamma;\rho)$, provided $\gamma>0,\ 0<\rho<1,\ z=0,1,2,\dots,$
where $M_n(z;\gamma;\rho)$ are the Meixner polynomials. (Rainville 1960) [8]

3. Substituting $\beta=-z,\gamma=-1; y=(1-e^{-\lambda})^{-1}$, we get,
$R_n(-z;-1;1,(1-e^{-\lambda})^{-1})=\left(\frac{1-e^{-\lambda}}{e^{\lambda}}\right)^{-n}\varphi_n(z;\lambda)$, where $\varphi_n(z;\lambda)$ are the Gottlieb polynomials. (Rainville 1960) [8]

4. Putting $\beta=-z,\gamma=N; y=(1-\rho)^{-1}$, we derive, $R_n(-z;-N;1,1-\rho)=(1-\rho)^n K_n(z;p,N)$, where $K_n(z;p,N)$ are the Krawtchouk polynomials. (Srivastava 1984) [11]

2. ASCENDING GENERATING RELATION

The Truesdell's *F*-equation

$$\frac{\partial}{\partial z}F(z,\alpha)=F(z,\alpha+1)$$

can be utilized to find a generating relation for $R_{\alpha+n}(\beta;\gamma;x,y)$ by using the ascending recurrence relation independently.

The polynomial $R_n(\beta;\gamma;x,y)$ satisfies the ascending recurrence relation

$$\frac{d}{dy}R_n(\beta;\gamma;x,y)=\frac{1}{y^2(x-y)}\big[(y-x)(\gamma+n)R_{n+1}(\beta;\gamma;x,y)$$
$$+y[(2n+\gamma-\beta)x-(\gamma+2n)y]R_n(\beta;\gamma;x,y)\big]. \tag{4}$$

Let $f(z,\alpha) = R_\alpha(\beta;\gamma;x,z)$ so that the equation (4) can be rewritten as

$$\frac{\partial}{\partial z} f(z,\alpha) = \frac{1}{z^2(x-z)}\big[(z-x)(\gamma+\alpha) f(z,\alpha+1)$$

$$+z[(2\alpha+\gamma-\beta)x-(\gamma+2\alpha)z]\, f(z,\alpha)\big]. \tag{5}$$

This equation is known as the *f*-type equation.

Symbolically, we write $$\frac{\partial}{\partial z} f(z,\alpha) = A(z,\alpha) f(z,\alpha) + B(z,\alpha) f(z,\alpha+1),$$

where $$A(z,\alpha) = \frac{1}{z^2(x-z)}\{z[(2\alpha+\gamma-\beta)x-(\gamma+2\alpha)z]\}$$

and $$B(z,\alpha) = \frac{1}{z^2(x-z)}\{(z-x)(\gamma+\alpha)\}.$$

Let us transform $f(z,\alpha)$ into $g(z,\alpha)$ as follows:

$$\frac{\partial}{\partial z} g(z,\alpha) = C(z,\alpha) g(z,\alpha+1). \tag{6}$$

Suppose

$$g(z,\alpha) = f(z,\alpha)\ \exp\left\{-\int_{z_0}^{z} A(z,\alpha)\, dz\right\}$$

$$= f(z,\alpha)\ \exp\left\{-\int_{z_0}^{z} \frac{1}{z^2(x-z)}\{z[(2\alpha+\gamma-\beta)x-(\gamma+2\alpha)z]\}\, dz\right\}$$

$$= f(z,\alpha)\left\{\frac{(x-z)^{-\beta} z^{\beta-2\alpha-\gamma}}{(x-z_0)^{-\beta} z_0^{\beta-2\alpha-\gamma}}\right\}.$$

On choosing $z_0 = \mu,$ we get

$$g(z,\alpha) = \mu^{2\alpha-\beta+\gamma}(x-\mu)^{\beta} z^{-(2\alpha-\beta+\gamma)}(x-z)^{-\beta} f(z,\alpha). \tag{7}$$

Therefore,

$$\frac{\partial}{\partial z} g(z,\alpha) = \mu^{2\alpha-\beta+\gamma}(x-\mu)^{\beta}\frac{\partial}{\partial z}\left[z^{-(2\alpha-\beta+\gamma)}(x-z)^{-\beta} f(z,\alpha)\right]$$

$$= -(\alpha+\gamma)\,\mu^{-2} g(z,\alpha+1).$$

Thus, the *g*-type equation is

$$\frac{\partial}{\partial z} g(z,\alpha) = -(\alpha+\gamma)\,\mu^{-2} g(z,\alpha+1). \tag{8}$$

Hence, $C(z,\alpha) = -(\alpha+\gamma)\mu^{-2}$ [using (6)]

$$= Z(z)A(\alpha),$$

where $Z(z) = -1$ and $A(\alpha) = (\alpha+\gamma)\mu^{-2}.$

Further, let us transform $g(z,\alpha)$ to $F(t,\alpha)$ as follows:

$$t=\int_{z_1}^{z} Z(v)\,dv=-z+z_1 \tag{9}$$

and $$F_0F(t,\alpha)=g(z,\alpha)\exp\left\{\int_{\alpha_0}^{\alpha}\log A(v)\,dv\right\}.$$

Now, if $\alpha_0=-\gamma$, then

$$\int_{-\gamma}^{\alpha}\log\left\{\mu^{-2}(v+\gamma)\right\}dv=\int_{-\gamma}^{\alpha}[\log\mu^{-2}+\log(v+\gamma)]\,dv$$

$$=\int_{-\gamma}^{\alpha}[\log\mu^{-2}+\log t]\,dt \qquad (\text{letting } t=v+\gamma,\ dt=dv)$$

$$=(\alpha+\gamma)\log\mu^{-2}+\log\frac{\Gamma(\alpha+\gamma)}{\sqrt{2\pi}}$$

[$\because\ \log\Gamma(x)=\int_{-\gamma}^{\alpha}\log t\,dt+\log\sqrt{2\pi}$ (Milne-Thompson (1965) [6] pp. 253)]

$$=\log\left[\frac{\Gamma(\alpha+\gamma)\mu^{-2(\alpha+\gamma)}}{\sqrt{2\pi}}\right].$$

Therefore, $$F_0F(t,\alpha)=g(z,\alpha)\exp\left[\log\frac{\Gamma(\alpha+\gamma)\mu^{-2(\alpha+\gamma)}}{\sqrt{2\pi}}\right].$$

Now, supposing $z_1=v$ and $F_0=\frac{1}{\sqrt{2\pi}}$, we get

$$F(t,\alpha)=g(z,\alpha)\Gamma(\alpha+\gamma)\mu^{-2(\alpha+\gamma)}g(v-t,\alpha)\,. \tag{10}$$

To show that $F(t,\alpha)$ does indeed satisfy the F-equation, let us determine $\frac{\partial}{\partial t}F(t,\alpha)$ as follows:

$$\frac{\partial}{\partial t}F(t,\alpha)=\Gamma(\alpha+\gamma)\mu^{-2(\alpha+\gamma)}\frac{\partial g(v-t,\alpha)}{\partial(v-t)}\cdot\frac{\partial g(v-t)}{\partial t} \tag{11}$$

$$=\Gamma(\alpha+\gamma+1)\mu^{-2(\alpha+\gamma+1)}g(v-t,\alpha+1).$$

Therefore, $$\frac{\partial}{\partial t}F(t,\alpha)=F(t,\alpha+1).$$

For obtaining generating relation, $F(t,\alpha)$ is expressed as follows:

$$F(t,\alpha)=\Gamma(\alpha+\gamma)\mu^{-2(\alpha+\gamma)}g(v-t,\alpha) \tag{12}$$

$$=\Gamma(\alpha+\gamma)\mu^{-2(\alpha+\gamma)}\mu^{2\alpha-\beta+\gamma}(x-\mu)^{\beta}(v-t)^{-(2\alpha-\beta+\gamma)}$$

$$(x-v+t)^{-\beta}f(v-t,\alpha)\quad[\text{using (7)}]$$

$$=\Gamma(\alpha+\gamma)\mu^{-\beta-\gamma}(x-\mu)^{\beta}(v-t)^{-(2\alpha-\beta+\gamma)}$$

$$(x-v+t)^{-\beta}R_{\alpha}(\beta;\gamma;x,v-t)\,.$$

Thus,

$$F(t+z,\alpha)=\Gamma(\alpha+\gamma)\mu^{-\beta-\gamma}(x-\mu)^{\beta}(v-t-z)^{-(2\alpha-\beta+\gamma)}$$
$$(x-v+t+z)^{-\beta}R_{\alpha}(\beta;\gamma;x,v-t-z) \tag{13}$$

and

$$F(t,\alpha+n)=\Gamma(\alpha+n+\gamma)\mu^{-\beta-\gamma}(x-\mu)^{\beta}(v-t)^{-(2\alpha+2n-\beta+\gamma)}$$
$$(x-v+t)^{-\beta}R_{\alpha+n}(\beta;\gamma;x,v-t). \tag{14}$$

Now, let us apply Truesdell's generating function theorem ((Truesdell 1948) [12] pp. 82, Theorem 14.1); if the function $F(t,\alpha)$ satisfies the *F*-equation and $F(t+z,\alpha)$ possesses a Taylor's series in powers of *z*, then this series may be expressed as

$$F(t+z,\alpha)=\sum_{n=0}^{\infty}\frac{z^n}{n!}F(t,\alpha+n) \tag{15}$$

which implies

$$\left(1+\frac{z}{x-v+t}\right)^{-\beta}\left(1-\frac{z}{v-t}\right)^{-(2\alpha-\beta+\gamma)}R_{\alpha}(\beta;\gamma;x,v-t-z)$$
$$=\sum_{n=0}^{\infty}\frac{(\alpha+\gamma)_n}{n!}(v-t)^{-2n}R_{\alpha+n}(\beta;\gamma;x,v-t)\,z^n.$$

Now, replacing $v-t$ by *y* and *z* by wy^2, we get the generating relation

$$(x-y)^{\beta}(1-yw)^{-\gamma+\beta-2\alpha}[x-y(1-yw)]^{-\beta}\ R_{\alpha}(\beta;\gamma;x,y(1-yw))$$
$$=\sum_{n=0}^{\infty}\frac{(\alpha+\gamma)_n}{n!}R_{\alpha+n}(\beta;\gamma;x,y)w^n. \tag{16}$$

3. DESCENDING GENERATING RELATION

Similarly, Truesdell's *G*-equation

$$\frac{\partial}{\partial z}G(z,\alpha)=G(z,\alpha-1)$$

can be used to derive generating relation for the set of polynomials $R_{\alpha-n}(\beta;\gamma;x,y)$ as follows:

The descending recurrence relation for $R_n(\beta;\gamma,x,y)$ is given by

$$\frac{d}{dy}R_n(\beta;\gamma;x,y)=\frac{n}{y(x-y)}[xR_n(\beta;\gamma;x,y)-y^2R_{n-1}(\beta;\gamma;x,y)]. \tag{17}$$

Let $f(z,\alpha)=R_{\alpha}(\beta;\gamma;x,y)$ so that

$$\frac{\partial}{\partial z}f(z,\alpha)=\frac{\alpha}{z(x-z)}[xf(z,\alpha)-z^2f(z,\alpha-1)]. \tag{18}$$

This equation is known as the *f*-type equation.

Symbolically, we write $\frac{\partial}{\partial z}f(z,\alpha)=A(z,\alpha)f(z,\alpha)+B(z,\alpha)f(z,\alpha-1),$

where $\quad A(z,\alpha)=\dfrac{\alpha x}{z(x-z)} \quad$ and $\quad B(z,\alpha)=\dfrac{-\alpha z}{x-z}$.

Now, let us transform $f(z,\alpha)$ into $g(z,\alpha)$ so that

$$\frac{\partial}{\partial z} g(z,\alpha)=C(z,\alpha)\, g(z,\alpha-1). \tag{19}$$

Therefore, suppose

$$\begin{aligned} g(z,\alpha) &= f(z,\alpha)\exp\left\{-\int_{z_0}^{z} A(v,\alpha)\,dv\right\} \\ &= f(z,\alpha)\exp\left\{-\int_{z_0}^{z} \frac{\alpha x}{z(x-z)}\,dz\right\} \\ &= f(z,\alpha)\left[\frac{z_0(x-z)}{z(x-z_0)}\right]^{\alpha}. \end{aligned}$$

On choosing $z_0=\mu,$ we get

$$g(z,\alpha)=\mu^{\alpha}(x-\mu)^{-\alpha} z^{-\alpha}(x-z)^{\alpha} f(z,\alpha). \tag{20}$$

Now,

$$\begin{aligned} \frac{\partial}{\partial z} g(z,\alpha) &= \frac{\partial}{\partial z}[\mu^{\alpha}(x-\mu)^{-\alpha} z^{-\alpha}(x-z)^{\alpha} f(z,\alpha)] \\ &= -\alpha\,\mu^{\alpha}(x-\mu)^{-\alpha} z^{-(\alpha-1)}(x-z)^{\alpha-1} f(z,\alpha-1) \\ &= -\alpha\,\mu(x-\mu)^{-1} g(z,\alpha-1). \qquad \text{[using (20)]} \end{aligned}$$

Thus, we have obtained *g*-type equation

$$\frac{\partial}{\partial z} g(z,\alpha)=-\alpha\,\mu(x-\mu)^{-1} g(z,\alpha-1). \tag{21}$$

Hence, $\quad C(z,\alpha)=-\alpha\,\mu(x-\mu)^{-1}$ [using (19)]

$$=Z(z)A(\alpha),$$

where $\quad Z(z)=-1$ and $A(\alpha)=\alpha\,\mu(x-\mu)^{-1}$.

Further, let us transform $g(z,\alpha)$ into $G(t,\alpha)$ as follows:

$$t=\int_{z_1}^{z} Z(v)\,dv \tag{22}$$

and

$$\begin{aligned} h(\alpha) &= \exp\left\{\int_{\alpha_0}^{\alpha+1} \log A(v)\,dv\right\} \\ &= \exp\left\{\int_{\alpha_0}^{\alpha+1} \log(v\mu(x-\mu)^{-1})\,dv\right\} \end{aligned}$$

Now, if $\alpha_0=0,$ we get

$$h(\alpha) = \exp\left\{\int_0^{\alpha+1} \log(v\mu(x-\mu)^{-1})\,dv\right\}$$

$$= \exp\left\{\log \frac{\sqrt{2\pi}\,\mu^{-(\alpha+1)}(x-\mu)^{\alpha+1}}{\Gamma(\alpha+1)}\right\}$$

$$= \frac{\sqrt{2\pi}\,\mu^{-(\alpha+1)}(x-\mu)^{\alpha+1}}{\Gamma(\alpha+1)}.$$

Thus, we have $G_0 G(t,\alpha) = h(\alpha) g(z,\alpha)$

$$= \frac{\sqrt{2\pi}\,\mu^{-(\alpha+1)}(x-\mu)^{\alpha+1}}{\Gamma(\alpha+1)} g(z,\alpha).$$

Now, choosing $z_1 = v$ and $G_0 = \sqrt{2\pi}$, we get

$$G(t,\alpha) = \frac{\mu^{-(\alpha+1)}(x-\mu)^{\alpha+1}}{\Gamma(\alpha+1)} g(v-t,\alpha). \tag{23}$$

To show that $G(t,\alpha)$ does indeed satisfy the *G*-equation, let us determine $\frac{\partial}{\partial t}G(t,\alpha)$ as follows:

$$\frac{\partial}{\partial t}G(t,\alpha) = \frac{\mu^{-(\alpha+1)}(x-\mu)^{\alpha+1}}{\Gamma(\alpha+1)} \frac{\partial g(v-t,\alpha)}{\partial(v-t)} \cdot \frac{\partial g(v-t)}{\partial(t)}$$

$$= \frac{\mu^{-\alpha}(x-\mu)^{\alpha}}{\Gamma(\alpha+1)} g(v-t,\alpha-1)$$

$$= G(t,\alpha-1). \tag{24}$$

Therefore, $\frac{\partial}{\partial t}G(t,\alpha) = G(t,\alpha-1).$

For obtaining the generating relation, let us suppose $G(t,\alpha)$ in the following form:

$$G(t,\alpha) = \frac{\mu^{-(\alpha+1)}(x-\mu)^{\alpha+1}}{\Gamma(\alpha+1)} g(v-t,\alpha) \quad \text{[using (23)]}$$

$$= \frac{\mu^{-(\alpha+1)}(x-\mu)^{\alpha+1}}{\Gamma(\alpha+1)} \mu^{\alpha}(x-\mu)^{-\alpha}(x-(v-t))^{\alpha}(v-t)^{-\alpha} f(v-t,\alpha)$$

$$= \frac{\mu^{-1}(x-\mu)}{\Gamma(\alpha+1)}(v-t)^{-\alpha}(x-v+t))^{\alpha} R_{\alpha}(\beta;\gamma;x,v-t). \tag{25}$$

Thus

$$G(t+z,\alpha) = \frac{\mu^{-1}(x-\mu)}{\Gamma(\alpha+1)}(v-t-z)^{-\alpha}(x-v+t+z)^{\alpha} R_{\alpha}(\beta;\gamma;x,v-t-z) \tag{26}$$

and

$$G(t,\alpha-n) = \frac{\mu^{-1}(x-\mu)}{\Gamma(\alpha-n+1)}(v-t)^{-\alpha+n}(x-v+t)^{\alpha-n} R_{\alpha-n}(\beta;\gamma;x,v-t). \tag{27}$$

Now, let us apply Truesdell's generating function theorem ((Truesdell 1948) [12], pp. 67, Theorem 2); by using Taylor's series $G(t+z,\alpha)$ can be expressed as

$$G(t+z,\alpha)=\sum_{n=0}^{\infty}\frac{z^n}{n!}G(t,\alpha-n) \tag{28}$$

which implies that

$$\left(1-\frac{z}{v-t}\right)^{-\alpha}\left(1+\frac{z}{x-v+t}\right)^{\alpha}R_\alpha(\beta;\gamma;x,v-t-z)$$
$$=\sum_{n=0}^{\infty}\frac{(-1)^n(-\alpha)_n}{n!}(v-t)^n(x-v+t)^{-n}R_{\alpha-n}(\beta;\gamma;x,v-t)\,z^n.$$

Now, replacing $(v-t)$ by y and z by $\dfrac{-w(x-y)}{y}$, we get the generating relation

$$y^\alpha(y-w)^\alpha(y^2-yw+xw)^{-\alpha}R_\alpha(\beta;\gamma;x,y^{-1}(y^2-yw+xw))$$
$$=\sum_{n=0}^{\alpha}\frac{(-\alpha)_n}{n!}R_{\alpha-n}(\beta;\gamma;x,y)\,w^n. \tag{29}$$

4. APPLICATIONS

The following generating relations are deduced from (16) and (29) using the conditions given in special cases:

1. $$\sum_{n=0}^{v}\frac{(-v-\alpha)_n}{n!}L_{v-n}^{(\alpha)}(x,y)c^n=\left(1-cy\right)^v L_v^{(\alpha)}\left(\frac{x}{1-cy},y\right),\ \left|c\right|<1.$$

2. $$\sum_{n=0}^{\infty}\frac{(1+v)_n}{n!}L_{v+n}^{(\alpha)}(x,y)b^n=(1-by)^{-\alpha-v-1}\exp\left(\frac{-bx}{1-by}\right)L_v^{(\alpha)}\left(\frac{x}{1-by},y\right),\ \left|b\right|<1.$$

3. $$\sum_{n=0}^{v}\frac{(-v)_n}{n!}(1-\rho)^n M_{v-n}(z;\gamma,\rho)c^n$$
$$=(1-c(1-\rho))^v M_v\left(z;\gamma,\left(\frac{c(1-\rho)-1}{c(1-\rho)-\rho^{-1}}\right)\right),\quad \left|c(1-\rho)\right|<1.$$

provided $\gamma>0,\ 0<\rho<1,\ z=0,1,2,...$

4. $$\sum_{n=0}^{\infty}\frac{(v+\gamma)_n}{n!}(1-\rho)^{-n}M_{v+n}(z;\gamma,\rho)b^n$$
$$=(1-b(1-\rho)^{-1})^{-z-\gamma-v}\left(1+b(1-\rho)\left\{1-(1-\rho)^{-1}\right\}^{-1}\right)^z$$
$$M_v\left(z;\gamma,\left(\frac{\rho(1-\rho)-b}{1-\rho-b}\right)\right),\quad \left|\frac{b}{(1-\rho)}\right|<1.$$

provided $\gamma>0,\ 0<\rho<1,\ z=0,1,2,...$

5. $$\sum_{n=0}^{v}\frac{(-v)_n}{n!}\left(\frac{1-e^{-\lambda}}{e^{\lambda}}\right)^n\phi_{v-n}(z;\lambda)c^n$$
$$=\left\{1-ce^{-\lambda}(1-e^{-\lambda})\right\}^v\ \phi_v\left(z;\log\left(\frac{c+e^{2\lambda}(1-e^{\lambda})^{-1}}{c+e^{\lambda}(1-e^{\lambda})^{-1}}\right)\right),\quad \left|\frac{c(1-e^{-\lambda})}{e^{\lambda}}\right|<1.$$

6. $$\sum_{n=0}^{\infty} \frac{(1+v)_n}{n!} \left(\frac{1-e^{-\lambda}}{e^{\lambda}} \right)^n \phi_{v+n}(z;\lambda) b^n = \left\{1-be^{-\lambda}(1-e^{-\lambda})^{-1}\right\}^{-z-1}$$

$$\left\{1+be^{2\lambda}(1-e^{\lambda})^{-1}\right\}^{z-v} \phi_v \left(z; \log\left(\frac{e^{\lambda}+be^{2\lambda}(1-e^{\lambda})^{-1}}{1+be^{2\lambda}(1-e^{\lambda})^{-1}} \right) \right), \left| \frac{be^{\lambda}}{(1-e^{\lambda})} \right| < 1.$$

7. $$\sum_{n=0}^{v} \frac{(-v)_n}{n!} (1-p)^{-n} K_{v-n}(z;p,N) c^n$$

$$= \left\{1-(1-p)^{-1}\right\}^{v} K_v \left(z; (p-cp(1-p)^{-1}), N \right), \quad \left| \frac{c}{(1-p)} \right| < 1.$$

provided $0<p<1,\ z=0,1,2,...,N.$

8. $$\sum_{n=0}^{\infty} \frac{(v-N)_n}{n!} (1-p)^n K_{v+n}(z;p,N) b^n$$

$$= (1-b(1-p))^{-z-v+N} (1+bp^{-1}(1-p)^2)^z K_v(z;(p+b(1-p)^2),N), \ \left| b(1-p) \right| < 1.$$

provided $0<p<1,\ z=0,1,2,...,N.$

5. CONCLUSIONS

We have obtained the generating relations for the polynomial set $R_n(\beta;\gamma;x,y)$ by the Truesdell method by giving a suitable interpretation to the index *n*. It is worth noting that the ascending and descending differential recurrence relations are used independently. It concludes that, each generating function, deduced its particular cases as applications to well known special functions.

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