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CALCULATION OF OPTIMUM GEAR RATIOS OF A TWO-STAGE BEVEL HELICAL GEARBOX

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ABSTRACT

This paper introduces a study on the calculation of optimum gear ratios of a two stage bevel helical gearbox. In this study, in order to determine the optimum gear ratios, an optimization problem was conducted in which the gearbox cross section area was chosen for the objective function. Besides, to weigh the effect of the input factor son the optimum gear ratios, a simulation experiment was designed and a computer program was built in order to perform the experiment. From the results of the experiment, the effect of the input factors on the optimum gear ratios were investigated and equations for determination of the optimum gear ratios were proposed.

Keywords: Gear Ratio, Optimum Gear Ratio, Optimum Gearbox Design, Bevel Helical Gearbox.

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1. INTRODUCTION

In gearbox design, the determination of the optimum gear ratios is very important. The reason of that is the gear ratios affect the dimension, the mass, and therefore the cost of agearbox. For example, Figure 1 shows the relation between the gear ratio of the second stage and the mass of gears of a two stage helical gearbox. It can be seen from the figure, with optimum gear ratio $u_2 = 2$, the gear mass (about 178 kg) is much more less than that when $u_2 = 5$ (about 242 kg).

Until now, for two stage bevel helical gearboxes, there are several studies on the calculation of the gear ratios. V.N. Kudreavtev et al [1] introduced a graph(Figure 2) for the determination ofthe gear ratio of the bevel gear set. However, for determining u_1, we must select the coefficient $\lambda_k c_k^3$ which is a large in value ($\lambda_k c_k^3$ = 10 to 60). Consequently, it is very complicated to find the gear ratio and it is impossible to getthe optimum value.G. Milou et al. [2] proposed a practical method in which the gear ratios were found in the tabulated form from practical data.Based on the data from gearbox factories, it was noted that the two-stage gearbox weight is minimum if the ratio betweenthe diameter of the bevel gear wheeland the center distance of the helical gear d_{ae2} / a_w ranged from 1.12 to 1.4 [2]. Based on that, the optimal gear ratios were proposed in the tabulated form.

The most useful method for determination of the optimum gear ratios is the model method. In this method, models for calculating the optimum gears are found for different objectives. The objective can be the minimum height of the gearbox [3], the minimum gearbox length [4] or the minimum cross-sectional dimension of the system [5].

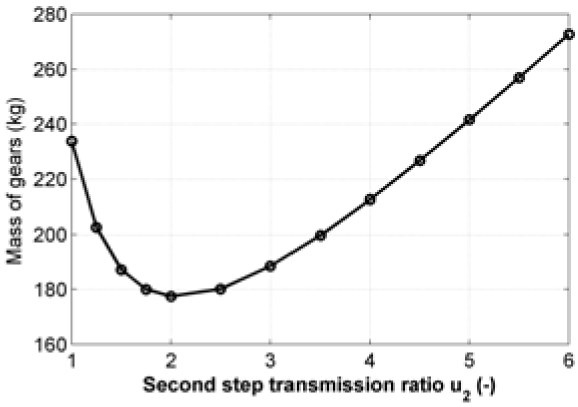

Figure 1: Partial ratios versusgear mass of a two-stage helical gearbox Calculation with $u_h = 10; T_1 = 60000 Nmm; \beta = 12^0; k_H = 1.3$

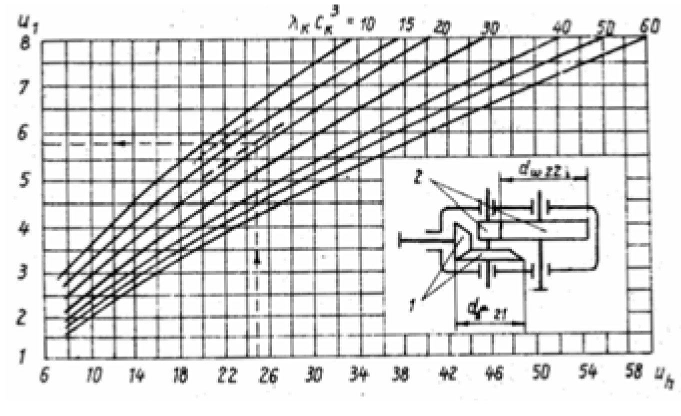

Figure 2 The gear ratio of the bevel gear set versus the total gearbox ratio [1]

This paper presents a study for calculation of the optimum gear ratios of a two stage bevel helical gearbox with the objective is the minimum gear box cross section area. Also, the effect of the input factors on the optimum gear ratios was investigated.

2. OPTIMIZATION PROBLEM

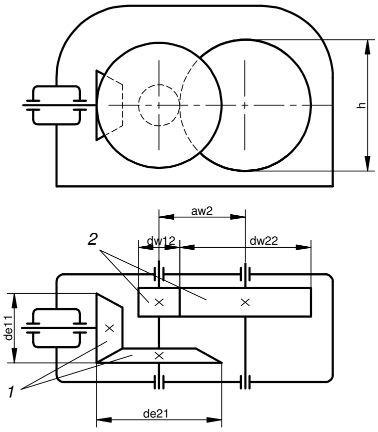

Figure 3: Calculation schema

From Figure 3, the gearbox cross section area can be calculated by:

$$A = L \cdot h \quad (1)$$

Where, L and h are determined as (see Figure 2):

$$L = d_{e21}/2 + a_{w2} + d_{w22}/2 \quad (2)$$

$$h = \max\left(d_{e21}, d_{w22}\right) \quad (3)$$

In the above equations, d_{e21} is the outer pitch diameter of the wheel of bevel gear set; a_{w2} and d_{w22} are the center distance and the pitch diameters of the helical gear set.

Thus, the optimization problem is defined as

$$\text{minimize } A = L \cdot h \quad (4)$$

With the following constraints

$$1 \le u_1 \le 6 \quad (5)$$

$$1 \le u_2 \le 9$$

From (2), (3), (4) and (5), it is clear that to solve the above optimization problem it is necessary to determine d_{e21}, a_{w2} and d_{w22}.

2.1. Determining the outer pitch diameter of the wheel

For a straight bevel gear set, the outer pitch diameter of the wheel is calculated by:

$$d_{e11} = 2 \cdot R_e / \sqrt{1+u_1^2} \quad (6)$$

In which, R_e is the external cone distance which is determined by[6]:

$$R_e = k_R \cdot \sqrt{u_1^2+1} \cdot \sqrt[3]{T_{11} \cdot k_{H\beta 1} / \left[\left(1-k_{be}\right) \cdot k_{be} \cdot u_1 \cdot \left[\sigma_H\right]^2\right]} \quad (7)$$

Where, k_R is the coefficient which depends on the gear material and the gear type; $k_R = 50$ ($MPa^{1/3}$) [6]; $k_{be} = 0.25 \ldots 0.3$ is the coefficient of the face width; $\left[\sigma_{H1}\right]$ is the allowable contact stress (MPa); u_1 is the gear ratio of the bevel gear set; $K_{H\beta 1}$ is the contact load ratio for pitting resistance of the bevel gear set. From the tabulated data in [6], the following regression equations were found for determination of $K_{H\beta 1}$ (with the coefficient of determination $R^2 = 1$):

$$K_{H\beta 1} = 0.25 \cdot k^2 + 0.2 \cdot k + 1.02 \qquad (8)$$

Wherein, $k = k_{be} \cdot u_1 / \left(2 - k_{be}\right)$.

From the condition of the moment equilibrium of the mechanic system which includes two gear sets we have:

$$T_{11} = T_{out} / \left(u_g \cdot \eta_{bg} \cdot \eta_{hg} \cdot \eta_b^3\right) \qquad (9)$$

In which, T_{out} is the output torque (Nmm); u_g is the total ratio of the gearbox; η_{bg} is the bevel gear transmission efficiency ($\eta_{hg} = 0.95\ldots0.97$ [6]); η_{hg} is the helical gear transmission efficiency ($\eta_{hg} = 0.96\ldots0.98$ [6]); η_b is the transmission efficiency of a pair of rolling bearing ($\eta_b = 0.99\ldots0.995$ [6]). Choosing $\eta_{bg} = 0.96$, $\eta_{hg} = 0.97$, $\eta_b = 0.992$ and substituting them into (9) gives:

$$T_{11} = 1.101 \cdot T_{out} / u_g \qquad (10)$$

Substituting $k_R = 50$ and (10) into (7) gets

$$R_e = 51.6296 \cdot \sqrt{u_1^2 + 1} \cdot \sqrt[3]{T_{out} \cdot k_{H\beta 1} / \left[\left(1 - k_{be}\right) \cdot k_{be} \cdot u_1 \cdot u_g \cdot \left[\sigma_H\right]^2\right]} \qquad (11)$$

2.2. Determining the center distance of the second stage

For the helical gear set, the center distance of the second stage a_{w2} is determined by [6]:

$$a_{w2} = k_m \cdot \left(u_2 + 1\right) \cdot \sqrt[3]{T_{12} \cdot k_{H\beta} / \left(\left[\sigma_H\right]^2 \cdot u_2 \cdot \psi_{ba2}\right)} \qquad (12)$$

In which, $K_{H\beta}$ is the contact load ratio for pitting resistance; $k_{H\beta} = 1.02 \div 1.28$ [6] and we can chose $k_{H\beta} = 1.1$; $\left[\sigma_H\right]$ is the allowable contact stress (MPa); In practice, $\left[\sigma_H\right] = 350\ldots410$ (MPa); k_m is the material coefficient; As the gear material is steel, k_m=43 [6]; ψ_{ba2} is coefficient of wheel face width of the helical gear set; $\psi_{ba2} = 0.3\ldots0.35$ [6];

Also, for this gearbox we have:

$$T_{out} = T_{12} \cdot \eta_{hg} \cdot \eta_b^2 \cdot u_2 \qquad (13)$$

Where, η_{hg} is helical gear transmission efficiency ($\eta_{hg} = 0.95\ldots0.97$ [6]); η_b is transmission efficiency of a pair of rolling bearing ($\eta_b = 0.99\ldots0.995$ [6]). Choosing $\eta_{hg} = 0.97$ and $\eta_b = 0.992$ gives

$$T_{12} = 1.0476 \cdot T_{out} / u_2 \qquad (14)$$

Substituting (14) and $k_{H\beta} = 1.1$ into (12) we have:

$$a_{w2} = 45.0814 \cdot (u_2 + 1) \cdot \sqrt[3]{T_{out} / \left([\sigma_H]^2 \cdot u_2^2 \cdot \psi_{ba2} \right)} \quad (15)$$

The pitch diameter of the second stage then is calculated by [6]:

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2 / (u_2 + 1) \quad (16)$$

2.3. Experimental work

For investigation of the effect of the input factors on the optimum gear ratios, a simulation experiment was designed and performed by a computer program. For this experiment, a 2-level full factorial design was selected. Also, Table 1 shows 5inputfactorswhich were chosen for the exploring. Hence, the design was arranged with $2^5 = 32$ number of experiments. To accomplish the experiment, a computer program was created based on equations (4) and (5). The various levels of input factors and the output responses (the optimum gear ratios of the second stage u_2) are presented in the Table 2.

Table 1: Input parameters

Factor	Code	Unit	Low	High
Total gearbox ratio	u_g	-	5	30
Coefficient of the face width of bevel gear set	Kbe		0.25	0.3
Coefficient of wheel face width of helical gear set	x_{ba2}	-	0.35	0.4
Allowable contact stress	AS	MPa	350	420
Output torque	T_{out}	Nmm	10^5	10^7

Table 2: Experimental plans and output response

StdOrder	RunOrder	CenterPt	Blocks	u_g	K_{be}	Xba2	AS (MPa)	Tout (Nm)	u_2
21	1	1	1	5	0.25	0.4	350	10000	2.93
2	2	1	1	30	0.25	0.35	350	100	4.89
30	3	1	1	30	0.25	0.4	420	10000	5.09
14	4	1	1	30	0.25	0.4	420	100	5.09
19	5	1	1	5	0.3	0.35	350	10000	2.73
17	6	1	1	5	0.25	0.35	350	10000	2.81
7	7	1	1	5	0.3	0.4	350	100	2.85
…									
18	31	1	1	30	0.25	0.35	350	10000	4.89
25	32	1	1	5	0.25	0.35	420	10000	2.81

3. RESULTS AND DISCUSSIONS

Figure 4 shows the main effect of each factors on the optimum gear ratios of the helical gear set u_2 (Figure 4). From the figure, it can be seen that u_2 increase significantly with the increase of the total gearbox ratio u_g. Also, it is affected by the coefficient of wheel face width of the bevel gear and the helical gear sets (k_{ba} and ψ_{ba2}). Besides, the allowable contact stress and the output torquedo not affect the optimum gear ratios.

The Pareto chart of the standardized effects is shown in Figure 5. As it is seen in the figure, the bars which represent parameters including the total gearbox ratio (factor A), the coefficient of wheel face width of the bevel gear (factor B) and the helical gear set(factors C) and the interactions between them cross the reference line. Therefore, these factors are statistically significant at the 0.05 level with the response model.

Figure 6 presents the Normal Plot of the standardized effects for u_2.From the figure, the total gearbox ratio u_g (factor A) is the most significant factor for u_2. Also, it and ψ_{ba2} (factor C) have a positive standardized effect for u_2. Besides, k_{be} (factor B) has a negative standardized effect for u_1 but they have a positive standardized effect for u_2.

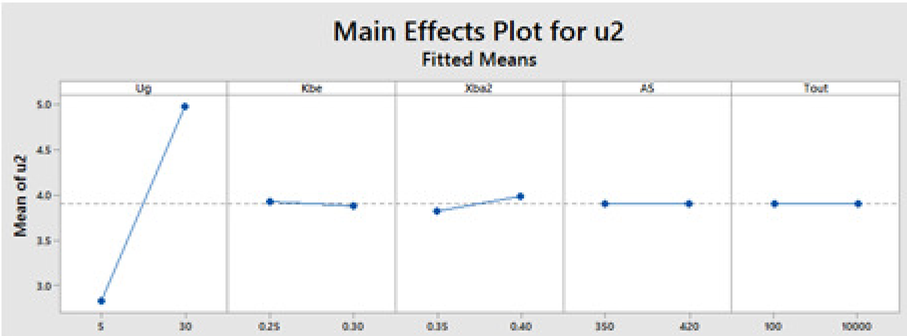

Figure 4. Main effects plot for u_2

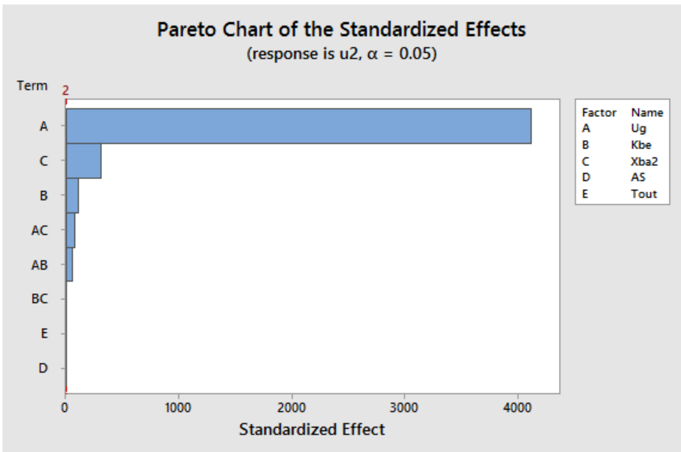

Figure 5. Pareto Chart of the Standardized Effects for u_2

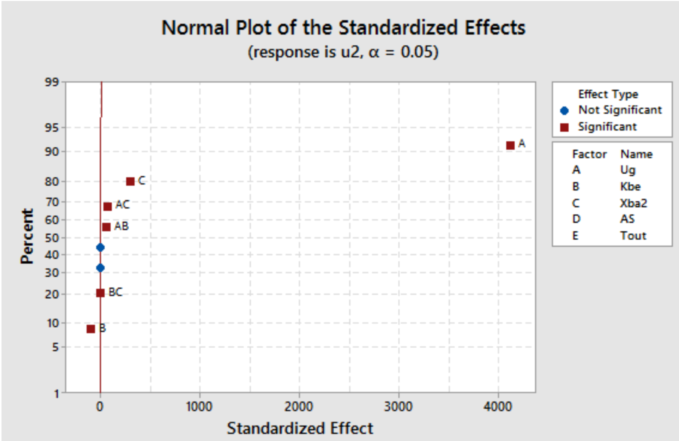

Figure 6. Normal Plot for u_2

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		3.90375	0.00026	14977.38	0.000	
Ug	2.14750	1.07375	0.00026	4119.62	0.000	1.00
Kbe	-0.052500	-0.026250	0.000261	-100.71	0.000	1.00
Xba2	0.157500	0.078750	0.000261	302.14	0.000	1.00
AS	0.000000	0.000000	0.000261	0.00	1.000	1.00
Tout	0.000000	0.000000	0.000261	0.00	1.000	1.00
Ug*Kbe	0.027500	0.013750	0.000261	52.75	0.000	1.00
Ug*Xba2	0.037500	0.018750	0.000261	71.94	0.000	1.00
Kbe*Xba2	-0.002500	-0.001250	0.000261	-4.80	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0014744	100.00%	100.00%	100.00%

Figure 7. Estimated Effects and Coefficients for u_2

Figure 7shows the estimated effects and coefficients for u_2.From the Figure, factors which have a significant effect on a response have P-values lower than 0.05 are the total gearbox ratio u_g, the coefficient of wheel face width of the bevel gear and the helical gear sets (k_{be} and ψ_{ba2}) and their interactions. Accordingly, the relation between the optimum gear ratios and the significant effect factors can be described by:

$$u_2 = 1.9072 + 0.0513 \cdot u_g - 1.07 \cdot k_{be} + 2.65 \cdot \psi_{ba2} + \\ +0.044 \cdot u_g \cdot k_{be} + 0.06 \cdot u_g \cdot \psi_{ba2} - 2 \cdot k_{be} \cdot \psi_{ba2} \quad (18)$$

Equation (18) fits the data very well because the adj-R^2 and pred-R^2 are in the high values (Figure 6). Thisequation isused to calculate the optimum gear ratio of the helical gear set u_2. After that, the optimum gear ratio of the bevel gear set is determined by $u_1 = u_g / u_2$.

4. CONCLUSIONS

A study on the determination of the optimum gear ratios of a two stage bevel helical gearbox for getting the minimum gearboxcross section area was performed. In this study, the effect of the input factors including the total gearbox ratio, the wheel face width coefficients of the bevel gear and helical gear sets, the allowable contact stress and the output torque was investigated. Also, equations for calculating the optimum partial gear ratios for getting the minimum gearbox cross section are a were proposed. Using these explicit equations, the optimum gear ratios can be determined simply.

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