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ON CONTINUITY CONDITIONS FOR THE PASSAGE OF SOUND THROUGH THE BOUNDARIES OF MEDIA

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ABSTRACT

It is shown that the Fresnel formulas for the coefficients of reflection and transmission of sound on the interface between the media obtained from the continuity conditions do not give an unambiguous answer concerning which wave parameters they represent: the oscillation speed or sound pressure. It is also shown that, in the classical solution of the problem of the obliquepassage of sound through the boundary of media, the continuity conditions are not satisfied. A physical model is proposed for calculating the passage of sound across the media boundaries, which takes into account the change in the widths of the sound beams as the angles of incidence and refraction of the waves change, so that the continuity condition is satisfied. The possibility is shown of applying the law of conservation of momentum and the law of conservation of kinetic energy in the problems of determining the coefficients of transmission and reflection of the oscillation speed of waves.

Keywords: Fresnel Formulas, Reflection And Passage of Sound, Continuity Of The Medium, The Law Of Conservation Of Momentum.

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1. INTRODUCTION

The problems of sound passing through the boundaries of media are solved on the basis of the condition of continuity of media on the boundary, which, according to [1, p.131], is formulated as follows: ... "the media should not move away from each other or penetrate into each other". In practice, this position is not verified, and the physical model of sound passing through the boundary of media, at its any point, is represented by the oscillation speed vectors of the incident, transmitted and reflected waves at any point of the boundary separating the media. Let us consider the validity of such an approach using the example of the passage of a plane harmonic wave through the plane boundary of two liquid or gaseous media.

2. METHODS

2.1. Problem 1. Normal incidence of sound.

The Fresnel formulas for the transmission coefficient αand the reflection coefficient β of the sound are obtained from the joint solution of the continuity equations for the oscillation speed and the sound pressure on the boundary of the media and have the following form:

$$\alpha = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1}, \quad (1)$$

$$\beta = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1}, \quad (2)$$

where$\rho$is the density of the medium, kg/m^3;

$c$is the wave propagation velocity in the medium, m/s.

Since one of the initial equations describes the continuity of the oscillation speed, and the other –the continuity of pressure, it remains unclear which of the indicated wave parameters the above coefficients present. Therefore, the scientific [2, p. 423-429], educational [1, p. 131-133], technical [3, p. 68-69] and reference [4, p. 566-568] literature contains opposite opinions concerning the meaning of sound transmission coefficients. This suggests that some additional actions are needed to identify the meaning of the coefficients represented by equations (1) and (2). For example, one can compare the results obtained by calculation with the actual values of the coefficients obtained from practice.

Table 1[5] shows the actual values of the sound transmission coefficients across the air and seawater boundary with respect to oscillation speed(column 2) and sound pressure (column 3), as well as the values calculated using Fresnel formulas (column 4). From the comparison of the above data, it can be seen that the transmission coefficients calculated by the Fresnel formulas refer to the sound pressure. The meaning of the reflection coefficients is not clarified. To have a clear idea of the meaning of the calculated coefficients, it is necessary to have one parameter of the wave in the initial two equations, for example, the oscillation speed. In this case, one could use the equation of the law of conservation of momentum containing velocity to the first power, and the equation of conservation of kinetic energy containing velocity to the second power.

Table 1 Comparison of the actual [1, p.134] and the calculated coefficients of transmission and reflection of sound from air to water and back with the normal incidence of a plane wave

No.	Actual coefficients of sound transmission through the air boundary (ρc =420 kg/m^{2}s) and sea water (ρc =1.5·10^{6}kg/m^{2}s) [1, p. 134]			Calculated coefficients obtained from continuity conditions (according to Fresnel formulas)		Calculated coefficients obtained from conservation laws	
1		with respect to velocity α_v	with respect to pressure α_p	of reflection $\beta = \frac{\rho_2 c_2 - \rho_1 c_1}{\rho_2 c_2 + \rho_1 c_1}$	of transmission $\alpha = \frac{2\rho_2 c_2}{\rho_2 c_2 + \rho_1 c_1}$	of reflection $\beta = \frac{\rho_1 c_1 - \rho_2 c_2}{\rho_1 c_1 + \rho_2 c_2}$	of transmission $\alpha = \frac{2\rho_1 c_1}{\rho_1 c_1 + \rho_2 c_2}$
	1	2	3	4	5	6	7
2	Fromairtowater	0.00057	1.99943	0.99943	1.99943	-0.99943	0.00057
3	From water to air	1.99943	0.00057	-0.99943	0.00057	0.99943	1.99943

To create a system of equations from the laws of conservation of momentum and kinetic energy in an acoustic medium, one can use the analogy with direct elastic impact of a body of mass m_1, moving with speed $v = 1$, on a resting body of mass m_2. The velocity obtained by the first body after the impact can be denoted by β, by the second body – by α. In this case, the equation of conservation of momentum can be written as follows:

$$m_1 v = m_1 v\beta + m_2 v\alpha\,, \qquad (3)$$

the equation of conservation of kinetic energy:

$$m_1 v^2/2 = m_1\beta^2/2 + m_2\alpha^2/2. \qquad (4)$$

The joint solution of equations (3) and (4) gives the value of the velocity of the first body after the impact:

$$\beta = \frac{m_1 - m_2}{m_1 + m_2}, \qquad (5)$$

the velocity of the second body equals

$$\alpha = \frac{2m_1}{m_1 + m_2}\,. \qquad (6)$$

As can be seen, the structures of formulas (1), (2) and (5), (6) are the same. Using the well-known relation between the speed of sound propagation c, the oscillation frequency f and the wavelengthλ, $\lambda = c/f$,one can represent formulas (1) and (2) in the following form, replacing there the order of indices by the order in formulas (5) and (6):

$$\alpha = \frac{2\rho_1\lambda_1}{\rho_1\lambda_1 + \rho_2\lambda_2}\,, \qquad (7)$$

$$\beta = \frac{\rho_1\lambda_1 - \rho_2\lambda_2}{\rho_1\lambda_1 + \rho_2\lambda_2}. \qquad (8)$$

Taking the cross-sectional area of the beams as unit, one can obtain the dimension of the terms of formulas (7) and (8) the same as in formulas (5) and (6). Thus, the presented formulas for the transmission and reflection coefficients of sound on the media boundary can be determined by the ratio of the masses of the parts of the media enclosed in the volumes limited by the wavelengths at the consideredfrequency and the unit areas of the cross sections of the incident, transmitted and reflected beams.

There are two objections to the application of the law of conservation of momentum in acoustics:

- the law is applicable to closed systems, which according to [6, p. 137] are defined as follows: "A system that includes all interacting bodies (so that none of the bodies of the system are affected by any other body except those included in the system) is called a *closed system*." Acoustic medium is not such.
- the total value of the oscillation speed and, consequently, the momentum for the complete wavelength is zero.

The first objection can be contrasted with the idea of a virtually-closed system consisting of two virtual bodies with the volumes limited by the lengths of the corresponding incident and transmitted waves. The cross-sectional area of bodies under the normal incidence of waves on the interface of media, based on the conditions of continuity, is the same. The volume of the first medium covered by the incident wave coincides with the volume covered by the reflected wave,because these waves propagate in the same medium and along the same line. The masses of bodies are obtained by multiplying the volumes by the density of the considered media and, in a closed system, they are represented by material points.

Such a virtual system is “formed” in an acoustic medium during the passage of a wave. The initial and final moments of this process are described, respectively, by the right-hand and left-hand sides of equation (3) of conservation of momentum.

The second objection can be refuted by introducing into the calculation the effective value of the oscillation speed.

Thus, the normal passage of sound through the medium boundary is approximated by the elastic interaction (direct collision) of two material points. Before the interaction, the material point representing the first medium has an effective value of the velocity equal to one; the velocity of the material point representing the second medium is zero. After the interaction, the material point representing the first medium acquires a velocity β, corresponding to the reflection coefficient, while the material point representing the second medium gets the velocity α, corresponding to the transmission coefficient.

The results of calculations by formulas (7) and (8) are presented in columns 6 and 7 of the table. There, these formulas are presented in the interpretation of the wave resistance ratios. Considering all the calculation results presented in the table, one can conclude that, to obtain formulas for the transmission and reflection of sound normally falling on the boundary of the media, with respect to the sound pressure, it is necessary to use formulas derived from the continuity conditions. For the formulas of coefficients with respect to the oscillation speed, it is necessary to use the equations of the laws of conservation of momentum and kinetic energy.

2.2. Problem 2. Oblique incidence of sound.

According to one of the versions [2, p.452], the Fresnel formulas for oblique incidence of plane waves on a flat interface between the media have the following form:

$$\alpha = \frac{2\rho_2 c_2 \cos\Theta_1}{\rho_2 c_2 \cos\Theta_1 + \rho_1 c_1 \cos\Theta_2}, \quad (9)$$

$$\beta = \frac{\rho_2 c_2 \cos\Theta_1 - \rho_1 c_1 \cos\Theta_2}{\rho_2 c_2 \cos\Theta_1 + \rho_1 c_1 \cos\Theta_2}, \quad (10)$$

where θ_1 and θ_2 are the angles of incidence and refraction on the media boundary.

Figure 1a shows a scheme of the normal incidence of a sound beam of arbitrary width *b* on the interface between two media. Let us assume that the other dimension of the crosssection of the beam is one. It can be seen that the width of the beam is the same in the incident, transmitted and reflected waves and is equal to the width of the trace of the beam OA on the boundary of the media. It is obvious that the entire sound field can be represented as completely covered by such beams without gaps and mutual overlap. This means that, under normal sound falling, continuity is observed at any point in the field.

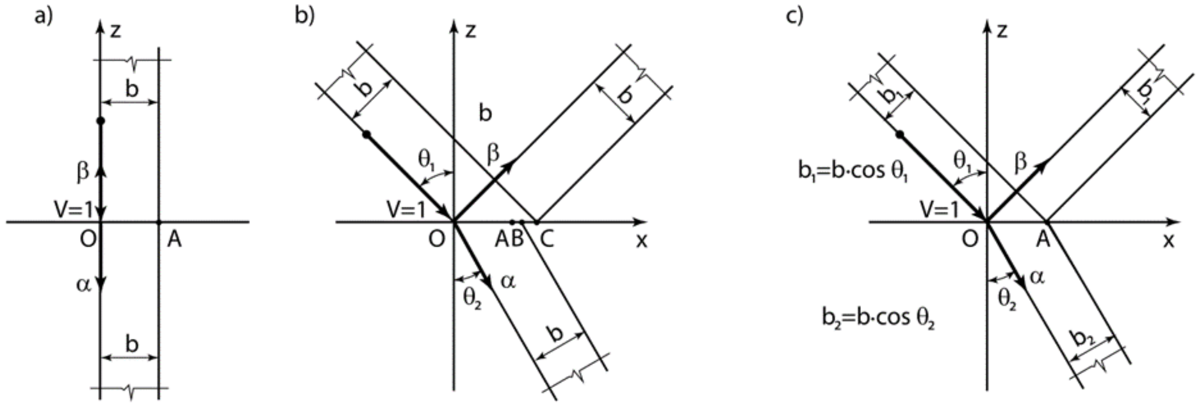

Figure1. Oblique passage of sound

Figure 1b presents a diagram of the oblique passage of sound, in which the incident, refracted (in accordance with the Snell's law) and reflected beams, according to formulas (9) and (10), have the same width *b*. In the figure, the segment OA means the width of the beam trace on the media boundary under normal falling of sound, the segment OB – the width of the trace of the transmitted beam at the refraction angle θ_2,and the segment OC – the width of the trace of the incident and reflected waves at the angle θ_1. As can be seen, the traces OB and OC of the beams on the media boundary under oblique sound fallingare different. *Consequently, the continuity conditions are not fulfilled and formulas (9) and (10) lose their meaning*.

Figure 1c shows the sound transmission scheme, for which the continuity conditions are fulfilled. In this case, the widths of all beams vary depending on their angle to the normal to the surface of the media boundary, but the traces of the beams on the media boundary are equal and coincide with the trace of the wave under the normal incidence. In these circumstances, writing the continuity conditions on the boundary of the media in the traditional way will not yield the necessary result. A new physical model of sound transmission is needed, which takes into account the change in the width of the beam as the angle of inclination changes [6] and, thus, the fulfillment of continuity condition is provided. Some attempts to create such a model were undertaken in [7-8].

Using the technique applied in solving the first problem, one can write the law of conservation of momentum in the following form:

$$\rho_1\lambda_1 cos\Theta_1\cdot v/cos\Theta_1 = \rho_1\lambda_1 cos\Theta_1\cdot\alpha/cos\Theta_1 + \rho_2\lambda_2\, cos\Theta_2\cdot\beta/cos\Theta_2. \quad (11)$$

Having made the necessary cancellations, one can obtain equation (3) of the law of conservation of momentum:

$$\rho_1\lambda_1\cdot v = \rho_1\lambda_1\cdot v\alpha + \rho_2\lambda_2\cdot v\beta.$$

That is, under the oblique incidence of sound, the equation of the law of conservation of momentum has the same form as with the normal incidence.

3. RESULTS

1. It is shown that the continuity conditions, on the basis of which the well-known solutions are constructed for the problems of the sound passage through the boundaries of media, are satisfied only under normal sound falling and not observed for oblique falling.

2. To solve the problem of the passage of sound through the boundaries of media, a physical model has been proposed, in which, unlike the classical one, the change in the widths of the incident, reflected and refracted beams is taken into account, thereby, the continuity conditions are satisfied. The model provides the possibility of applying the law of conservation of momentum in the problems of determining the coefficients of transmission and reflection of the oscillation speed of sound waves under different angles of their falling on the boundary of media.

3. It is shown that, for various angles of sound incidence on the boundary of the media, the transmission and reflection coefficients have the same values as for the normal incidence.

CONCLUSION

The obtained results make it possible to significantly simplify the solution of applied problems of the passage of sound and vibrations by applying only linear computational models in architectural and technical acoustics, as well as in seismology.

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