STUDY THE STEADY-STATE AND DYNAMIC PROBLEMS OF THE ROTATING BLADES

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ABSTRACT
The most failures in the rotating blades occur due to the high centrifugal forces (high speed), these centrifugal forces will produce a high stresses and deformations. In some cases, the generated stresses exceed the allowable stresses of the rotating blades; this situation will lead to reduce the operation life of the rotating blade and eventually failure will occur. A finite element model of the rotating blade has been developed using super-parametric shell element to obtain the stresses and deformations of the rotating blade. It was built a finite element program to achieve the numerical analysis using Fortran90 language. The results presented the stresses and deformations under the study-state condition and the natural frequency of the rotating blade.

Keywords: Rotating blades; Stresses and deformations; Finite element analysis
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1. INTRODUCTION
The rotating blade is essential part in many machines that used in wide field of engineering such as wind turbine, compressor, helicopter, axial fan, etc. The rotating blades are subjected mostly to fatigue stresses this which lead blade failure after short time of operation. Therefore, the estimation of the deformations and stresses with high precision is considered fundamental issue for mechanical engineers design. This accurate estimation of stresses will be very useful to obtain a successful design with long life operation. Figure 1 shows two different models of the damaged wind turbine blades due to the high stresses and vibration.
Study the Steady-State and Dynamic Problems of the Rotating Blades

Yin et al. [1] proposed a new methodology to study the blades of wind turbine based on perturbation methods. The Coriolis effect on the dynamic effect was investigated in details using different methods. It was applied three different methods of perturbation in their work, they are: the standard perturbation method and the other methods are which proposed by H. C. Hu and S. H. Chen, respectively. The obtained results of the blade of wind turbine showed that the high accuracy results can be obtained when use the two improved methods and less accuracy appeared when use the standard perturbation method.

Ul-Hamid et al. [2] studied the failure problem due to the vibration in the blades of the multi-stage low pressure centrifugal air compressor. Through the experimental investigation, it was found that the damage occurred in the 3rd stage of the impeller blade which used in a three-stage compressor with two inter-stage cooling tanks. It was found that the main source of the high stresses is high value of the hardness of impeller material which exceeds the allowable limit. The other source of the failure in the impeller blade is corrosion fatigue. Also, the high stresses which appeared due to the intensive vibration that existing in the compressor system, this kind of stresses lead to the fatigue failure.

Fernandez et al. [3] computed the stresses and strain of the wind turbine blade (5 MW) under heavy duty conditions. They achieved computational fluid dynamics (CFD) based Blade Element Momentum (BEM) to simulate the wind turbine blade. This research paper developed a new approach to study coupling problem in the wind blade to compute the stresses and deformations applying different load conditions. It was calculated the distribution of pressure on the blade due to the wind based on the aero-elastic method. The developed procedure reduced the analysis time needed to study the stresses and deformations of the wind blade.

Acar and Feeny [4] studied the dynamic problem of the wind turbine blade based on the bend-bend-twist coupled vibrations. They used the beam factors to obtain the forms of the potential and kinetic energy. It was applied the Lagrange's method to obtain the equation of motion. In their work the centrifugal effect was included in the stiffness matrix. It was found that the value of the stiffness of the structure is very sensitive to the hub angle. The results presented the natural frequencies and mode shapes taken into consideration the influence of the speed of rotation when it’s constant. It was found that the effects of the centrifugal and gravitational effects are considerable for the long blades and this effects decreases when the long of the blade decrease too.

Zarrinzadeh et al. [5] investigated the characteristics of the free vibration of rotating tapered beams which axially functionally graded under six different working conditions using finite element technique. It was used the beam element theory based on two-noded element, in order to obtain the displacement functions. The obtained results based on the developed approach were very accurate, where this approach took into consideration the influences of
the centrifugal force, type of the cross-sectional area and variation of the material properties. Other parameters were investigated, such as taper ratio, non-homogeneity of the material, speed of rotation, radius of the hub and mass at the tip.

Ronge et al. [6] studied the dynamic behavior of the helicopter rotor system works under heavy duty conditions. It was assumed that the environment leading is where unsteady to severe vibratory loads. It was found that the damage occurred in some composite rotor blades when it subjected to the repeated load. Also, it was investigated the dynamic behavior of the damaged composite rotor blades. This study is very important to fill the gap about the Structural Health Monitoring to diagnose the damage in the early stage of the helicopter. The experimental work covers a wide range of speed of rotation. It was measured the strains in different directions for the damaged and undamaged rotating blades. Furthermore, the vibration characteristics were measured experimentally for the damaged and undamaged rotating and non-rotating blades.

Jalali and Shahriari [7] achieved the stress analysis elastically of the rotating annular disk where the material of the disk functionally graded material (FGM). They assumed that the thickness of the desk, modules of the elasticity, density vary with radial direction according to the power-law function. They computed numerically the stresses (radial and circumferential) and deformations of the rotating disk under different conditions (free-free, clamped-clamped, and clamped-free). It was found that the values of the radial stresses reduced when use the rotating disk made from FG material, but the radial deformation increased.

Shang et al. [8] built mathematical model based on Timoshenko beam to study the steady-state and dynamic behavior of the pre-twisted composite rotor blades. The non-linearity of geometry and variation of the properties of materials in the directions were taken into consideration in their model. It was presented the generalized Timoshenko strain energy based on the equilibrium equations and the second-order strain energy. The nonlinearity problem was solved based on the Hamilton’s principle. The analytic results were compared with experimental results, and the differences between them were very small. It was found that the length of chord ratio of the blade is a significant factor on the results of the steady-state deformation and the natural frequencies of the pre-twisted composite rotor blade.

Aksencer and Aydogdu [9] developed a new model of the rotating laminated composite beam including point mass to study the vibration characteristics based on the Ritz method with algebraic polynomials. The boundary condition was assumed in their analysis which is clamped-free. They used the 1st and 3rd order of the shear deformation theories with the classic beam theory to obtain the mathematical formulation of the developed model. Many perimeters were studied such as the effect of mass ratios (mass ratio= attached mass/beam mass), speed of rotation, thickness ratio on the behavior of the rotating beam.

Zhang et al. [10] Built a new numerical model to study the coupling problem (fluid-structure) for the rotating blades (large-scale offshore, NREL 5MW). The vibration characteristics of the blades were found due to interaction of the coupling (fluid-structure). Ansys WORKBENCH software was used to build and analysis the coupling problem, in addition to the study the dynamic stability of the structure applying different values of speed of rotation. It was found that the most effective factors on the behavior and performance of the rotating blades are the fluid-structure interaction and the speed of rotation. The other important point was found that the values of the displacement and stresses increases dramatically when the frequency of the wind close to the fundamental natural frequency of the blade.

In this paper, the finite element program was developed and coded by using Fortran90 to study the steady state and dynamic behavior of the rotating blades. The results presented the radial deformations and Von-Mises stresses under steady-state condition and the natural
frequencies. The effects of the speed of rotation, thickness and skew angle on the deformations and stresses were investigated deeply.

2. FINITE ELEMENT FORMULATION

There are many difficulties to compute the stresses and deformations of the rotating blades, these difficulties come from the complexities in the geometry of the blades owing to the asymmetry in the cross-section, skew angle and pre-twist angle. In the most researchers in this field proved that the results of the stresses and deformations based on the plate and shell analyses for blades are very accurate compared with other one. Therefore, it can be considered that the shell theory with finite element is a useful tool to study the steady and dynamic behaviors of the rotating blades.

In this section, the finite element formulation based on the super-parametric parabolic shell element [11] is presented, Figure 2 illustrated the Shell element with 8-nodes. It was derived the equations of motion based on the Lagrange’s equation, and the prescribed displacement field corresponding to the super-parametric shell element which used to calculate the various derived matrices. The generic displacements in terms of nodal displacements are [12]:

\[
\begin{bmatrix}
u \\
\end{bmatrix} = \sum_{i=1}^{8} f_i \begin{bmatrix}
u_i \\
\end{bmatrix} + \sum_{i=1}^{8} f_i \xi_i \frac{h}{2} \mu_i \begin{bmatrix}\alpha_i \\
\end{bmatrix}
\]

(1)

Where \(f_i\) is the shape function and the symbol \((\mu_i)\) represents in the following Matrix:

\[
\mu_i = \begin{bmatrix}
-l_{2i} & l_{1i} \\
-m_{2i} & m_{1i} \\
-n_{2i} & n_{1i}
\end{bmatrix}
\]

(2)

In the super-parametric shell element, there are six types of non-zero strains in the global directions as follows [12]:

\[
\varepsilon = \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix} = \begin{bmatrix}
u_x \\
v_y \\
w_z + u_x \\
u_y + v_x \\
v_z + w_y \\
w_x + u_z
\end{bmatrix}
\]

(3)

The form of the stiffness matrix of the element that involving the integrals over the volume of the element is [12],

\[
\int [S] dx dy dz
\]

(4)

Where the matrix \([S]\) is a function of the coordinates. In the stiffness matrix [12]:

\[
[S] = B^T E B
\]

(5)

Where
\[ \varepsilon = B q \]  
(6)

Where \( B \) is the stress-displacement matrix and \( q \) is the nodal displacement.

The stress-strain relationships in the local directions for the orthotropic or isotropic materials is,

\[ \sigma' = E' \varepsilon' \]  
(7)

Or

\[
\begin{bmatrix}
\sigma'_{x'} \\
\sigma'_{y'} \\
\sigma'_{z'} \\
\tau'_{x'y'} \\
\tau'_{y'z'} \\
\tau'_{z'x'}
\end{bmatrix} =
\begin{bmatrix}
E_{x'x'} & E_{x'y'} & 0 & 0 & 0 & 0 \\
E_{x'y'} & E_{y'y'} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & G_{x'y'} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{G_{y'z'}}{k} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{G_{z'x'}}{k}
\end{bmatrix}
\begin{bmatrix}
\varepsilon'_{x'} \\
\varepsilon'_{y'} \\
\varepsilon'_{z'} \\
\gamma'_{x'y'} \\
\gamma'_{y'z'} \\
\gamma'_{z'x'}
\end{bmatrix}
\]  
(8)

The factor \( k \) which included in the last two terms of the shear is assumed equal to (1.2) and this factor have a positive effect to improve the accuracy of the results of the shear displacement. It can be written the formulation of the element stiffness matrix as follows,

\[
[k] = \int_{-1}^{1} \int_{-1}^{1} B^T E B \frac{d\xi d\zeta}{J} d\xi d\zeta
\]  
(9)

In the equation (9), \( B \) and \( J \) are function of \( \xi, \zeta \) and \( \zeta \). It can be get the final formulation of the stiffness after apply the integration on the equation (9) through the thickness of the element [12],

\[
[k] = \int_{-1}^{1} \int_{-1}^{1} (B_a + \zeta B_b)^T E (B_a + \zeta B_b) \frac{d\xi d\zeta}{J} d\xi d\zeta
\]  
(10)

Where the size of the matrices \( B_a \) and \( B_b \) are 6*40, while the other matrices contain just the terms which multiplied by \( \zeta \). In order to obtain the stiffness from Eq. (10), it should be to use the numerical approach. The mass matrix of the super-parametric shell element is [12],

\[
[M] = \int_{-1}^{1} \int_{-1}^{1} \rho \frac{d\xi d\zeta}{J} d\xi d\zeta
\]  
(11)

Where \([M_0]\) is,

\[
[M_0] = \int_{-1}^{1} \int_{-1}^{1} \rho \frac{d\xi d\zeta}{J} d\xi d\zeta
\]  
(12)

The equation of equilibrium for a system in motion as follows is [13]:

\[ [M]\ddot{U} + [C]\dot{U} + [K]U = \{R\} \]  
(13)

Where \([M], [C]\) and \([K]\) are the mass matrix, damping matrix and stiffness matrix, \( \{R\} \) is the external load vector (in our research paper the load is the centrifugal force). \( \{U\}, \{\dot{U}\} \)
and \{ \bar{U} \} are the displacement vector, velocity vector and acceleration vector of the finite element assemblage.

In this research paper, finite element program was coded with Fortran90. This program can handle with steady-state problem and dynamic problem (Modal Problem) of the rotating blades. Figure 3 shows the flow chart of the developed finite element program. Figure 4 presents the finite element model of the rotating blade, where for this model the accuracy of the results were tested against the number of selected element.

![Figure 2 Shell element with 8-nodes](image)

![Figure 3 The flowchart of the finite element program](image)
3. RESULTS AND DISCUSSIONS

This section presents the numerical results which obtained from the developed finite element program. The results were investigated the most important factors affected the deformations and stresses of the rotating blade such speed of rotation, thickness and skew angle.

Figures (5 and 6) show the variations of the radial deformations and Von-Mises stresses with the speed of rotation for different aspect ratios (l/b= 3, 4 and 5). It can be seen from these figures that the deformations and stresses increase dramatically when the speed of rotation increases too. The reason to obtain these results is the high increasing in the magnitude of the centrifugal force. Where, the centrifugal force is a function of the speed of rotation. Also, it’s clear that the deformations and stresses increase when the aspect ratio increases too. This happened due to increases in the mass of the blade, this increases in the mass will increase the amount of the centrifugal force. This increment in the centrifugal force leads to increase the deformations and stresses.

The effect of thickness of the blade on the radial deformations and Von-Mises stresses can be seen in Figures (7 and 8). From results, it’s clear that the deformations and stresses decreased when the thickness increased. The increases of the blade’s thickness lead to increase the structure stiffness of blade that yields to decrease the deformations and stresses. Also, the highest deformations and stresses appeared when use the high aspect ration (l/b=5) and lowest deformation and stresses when use the low aspect ration (l/b=3).

Figures (9 and 10) show the radial deformations and Von-Mises stresses with skew angle for different aspect ratios. It can recognized, that the deformations and stresses increased with skew angle, and the maximum values occurred when the skew (40˚). After this, the deformations and stresses decreased. The effect of skew angle appeared just on the stresses, but the effect of skew angle on the deformations is very small and can be neglected.

Figure 11 shows how it’s change the values of the natural frequencies with increasing of thickness of the blade. It can be seen, that the values of the natural frequencies increased when the thickness of the blade increased too, this is because the stiffness of the blade increase which lead to increase the value of the natural frequency. The other point, the bade which have the low aspect ratio has the higher value of the nature frequency higher, and vice versa.
Study the Steady-State and Dynamic Problems of the Rotating Blades

**Figure 5** The variation of the radial deformation with speed of rotation for different aspect ratios (Skew angle=15° and Pre-twist angle =15°)

**Figure 6** The variation of the Von-Mises stresses with speed of rotation for different aspect ratios (Skew angle=15° and Pre-twist angle =15°)

**Figure 7** The variation of the radial deformation with thickness for different aspect ratios (Skew angle=15° and Pre-twist angle =15°)
Figure 8 The variation of the Von-Mises stresses with thickness for different aspect ratios (Skew angle=15° and Pre-twist angle =15°)

Figure 9 The variation of the radial deformation with skew angle for different aspect ratios (Speed of rotation = 200 rad/s and Pre-twist angle =15°)

Figure 10 The variation of the Von-Mises stresses with skew angle for different aspect ratios (Speed of rotation = 200 rad/s and Pre-twist angle =15°)
4. CONCLUSIONS

In this work, the finite element program was coded by using Fortran90. This program has the ability to analyze the deformations and stresses of the rotating blade. It can be also, study different design factors such skew angle, pre-twist angle, thickness etc. In addition to this, it can be study different configurations of blades (e.g. cylindrical, tapered).

It was found that main factor affected the deformations and stresses are the speed of rotation where the deformations and stresses increase considerably with speed of rotation. While the deformations and stresses decreased when the thickness of the blade increases due to the increasing in the blade’s stiffness. The natural frequency has different behavior, where it’s increases when the thickness of the blade increases too. The skew angle affected strongly the stresses and peak of the stresses occurred when the skew angle is (40˚). While, the skew angle has no effect on the deformations as seen in the results section.

There are many other factors affect the behavior and performance of the rotating blades such thermal effect or dynamic effect. This paper is preliminary analysis of rotating blades and it will be followed by other research papers which investigate deeply the rotating blades under heavy duty conditions.

REFERENCES


