ANALYSIS OF RING STIFFENED CYLINDRICAL SHELLS USING FUNCTIONALLY GRADED MATERIAL

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ABSTRACT

The cylindrical shells structures play an important role for underwater applications, space vehicles and aircrafts in terms of its buckling, stiffness, strength and weight etc. Hence the importance of lightweight and high strength materials such as aluminum alloys, titanium alloys and composite materials have become significant. Functionally Graded Material (FGM) belongs to class of advanced material characterized by variation in properties as the dimension varies.

In this thesis finite element analysis is performed for a ring – stiffened cylindrical shell consisted of functionally graded material (FGM) subjected to hydrostatic pressure. Mathematical correlations are performed to determine the material properties of functionally graded material with metal Titanium using Ceramic as interface zone for each layer up to 10 layers. FGM’s are considered for volume fractions of K=2. Structural analysis, Modal analysis is done to compare different materials Aluminum alloy, Titanium alloy and FGM.

Key words: Aluminum alloy, Titanium alloy, FGM, K=2, Cylindrical Shells.


http://www.iaeme.com/IJMET/issues.asp?JType=IJMET&VType=8&IType=7

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1. INTRODUCTION

Shell structures play an important role for underwater applications, space vehicles and aircrafts in terms of its buckling, stiffness, strength and weight etc. Also the importance of exploration of lightweight and high strength materials such as alloys, titanium alloys and composite materials have become significant. It is assumed that the edges are effectively supported by ring frames, bulkheads or end closures. Stiffened circular cylindrical shells have to be dimensioned against several buckling failure modes. The methods are to be considered as semi-empirical.

The reason for basing the design on semi empirical methods is that the agreement between theoretical and experimental buckling loads for some cases has been found to be non-existent. This discrepancy is due to the effect of geometric imperfections and residual stresses in fabricated structures. Actual geometric imperfections and residual stresses do not in general appear as explicit parameters in the expressions for buckling resistance. This means that the methods for buckling analysis are based on an assumed level of imperfections.

Very few experimental data are available on stiffened cylindrical shells that fail from hydrostatic external pressure at the low values of pressure of interest in aerospace structures. Those data that are available were obtained on cylinders with widely spaced stiffeners so that local buckling of the skin preceded general failure of the cylinder. More desirable structures have closely spaced stiffeners so that general failure is expected to occur before, or perhaps at, the load for which skin buckling is expected. The purpose of the present investigation is to present test results for a hydrostatically loaded, 10-foot-diameter (3-meter), ring-reinforced cylinder and to compare the results with those predicted with the use of design equations for such structures. In addition, shortcomings in contemporary design analyses are discussed.

2. MATERIAL PROPERTY CALCULATIONS FOR FGM

2.1. For Young’s Modulus

For \( k=2; \ z=1 \)
\[
E(Z) = (E_t - E_b)(z/h+1/2)^k + E_b
\]

For \( k=2; \ z=-1 \)
\[
E(Z) = (E_t - E_b)(z/h+1/2)^k + E_b
\]

2.2. Material Properties

For Young’s Modulus

Material properties

Top material: ceramic \((E_T=380000\text{MPa})\)
Bottom material: Aluminum \((E_b=68000\text{ MPa})\)

For \( k=2; \ z=1 \)
\[
E(Z_1) = (E_t - E_b)(z/h+1/2)^k + E_b
\]
\[
= (380000-68000)(1/5+1/2)^2+68000
\]
\[
= (312000)(0.49)+68000
\]
\[
= 152880+68000
\]

\[ E(Z_1) = 220880 \]

For \( k=2; z=2 \)
\[ E(Z_2) = (E_t - E_b)(z/h + 1/2)^k + E_b \]
\[ = (380000-68000)(2/5 + 1/2)^2 + 68000 \]
\[ = (312000)(0.81) + 68000 \]
\[ = 252720 + 68000 \]
\[ E(Z_2) = 320720 \]

For \( k=2; z=3 \)
\[ E(Z_3) = (E_t - E_b)(z/h + 1/2)^k + E_b \]
\[ = (380000-68000)(3/5 + 1/2)^2 + 68000 \]
\[ = (312000)(1.21) + 68000 \]
\[ = 377520 + 68000 \]
\[ E(Z_3) = 445520 \]

For \( k=2; z=4 \)
\[ E(Z_4) = (E_t - E_b)(z/h + 1/2)^k + E_b \]
\[ = (380000-68000)(4/5 + 1/2)^2 + 68000 \]
\[ = (312000)(1.69) + 68000 \]
\[ = 527280 + 68000 \]
\[ E(Z_4) = 595280 \]

For \( k=2; z=5 \)
\[ E(Z_5) = (E_t - E_b)(z/h + 1/2)^k + E_b \]
\[ = (380000-68000)(5/5 + 1/2)^2 + 68000 \]
\[ = (312000)(2.25) + 68000 \]
\[ = 702000 + 68000 \]
\[ E(Z_5) = 770000 \]

For \( k=2; z=-1 \)
\[ E(Z_{-1}) = (E_t - E_b)(z/h + 1/2)^k + E_b \]
\[ = (380000-68000)(-1/5 + 1/2)^2 + 68000 \]
\[ = (312000)(0.09) + 68000 \]
\[ = 28080 + 68000 \]
\[ E(Z_{-1}) = 96080 \]

For \( k=2; z=-2 \)
\[ E(Z_{-2}) = (E_t - E_b)(z/h + 1/2)^k + E_b \]
\[ = (380000-68000)(-2/5 + 1/2)^2 + 68000 \]
\[ = (312000)(0.01) + 68000 \]
\[ = 3120 + 68000 \]
\[ E(Z_{-2}) = 71120 \]
For \( k=2; \ z=-3 \)
\[
E(Z_{-3}) = (Et-Eb)(z/h+1/2)^k + Eb \\
= (380000-68000)(-3/5+1/2)^2 + 68000 \\
= (312000) \times (0.01) + 68000 \\
= 3120 + 68000 \\
E(Z_{-3}) = 71120
\]

For \( k=2; \ z=-4 \)
\[
E(Z_{-4}) = (Et-Eb)(z/h+1/2)^k + Eb \\
= (380000-68000)(-4/5+1/2)^2 + 68000 \\
= (312000) \times (0.09) + 68000 \\
= 28080 + 68000 \\
E(Z_{-4}) = 96080
\]

For \( k=2; \ z=-5 \)
\[
E(Z_{-5}) = (Et-Eb)(z/h+1/2)^k + Eb \\
= (380000-68000)(-5/5+1/2)^2 + 68000 \\
= (312000) \times (0.25) + 68000 \\
= 78000 + 68000 \\
E(Z_{-5}) = 146000
\]

2.3. For Densities

Material Properties:
Ceramic(\( \rho_t=0.00000396 \text{Kg/mm}^3 \))
Aluminum(\( \rho_b=0.0000026989 \text{Kg/mm}^3 \))

For \( k=2; \ z=1 \)
\[
\rho(Z)= (\rho_t-\rho_b)(z/h+1/2)^k + \rho_b \\
= (0.00000396-0.0000026989)(1/5+1/2)^2 + 0.0000026989 \\
= 1.2611 \times 10^{-6} (0.49) + 0.0000026989 \\
= 0.000003307739 \text{Kg/mm}^3
\]

For \( k=2; \ z=2 \)
\[
\rho(Z)= (\rho_t-\rho_b)(z/h+1/2)^k + \rho_b \\
= (0.00000396-0.0000026989)(2/5+1/2)^2 + 0.0000026989 \\
= 1.2611 \times 10^{-6} (0.81) + 0.0000026989 \\
= 0.000003720391 \text{Kg/mm}^3
\]

For \( k=2; \ z=3 \)
\[
\rho(Z)= (\rho_t-\rho_b)(z/h+1/2)^k + \rho_b \\
= (0.00000396-0.0000026989)(3/5+1/2)^2 + 0.0000026989 \\
= 1.2611 \times 10^{-6} (1.21) + 0.0000026989 \\
= 0.000004164936 \text{Kg/mm}^3
\]
For \( k=2; \ z=4 \)
\[ \rho(Z) = (\rho_t - \rho_b)(z/h+1/2)^k + \rho_b \]
\[ = (0.00000396 - 0.0000026989)(4/5+1/2)^2 + 0.0000026989 \]
\[ = 1.2611 \times 10^{-6} (1.69) + 0.0000026989 \]
\[ = 0.000004830159 \text{ Kg/mm}^3 \]

For \( k=2; \ z=5 \)
\[ \rho(Z) = (\rho_t - \rho_b)(z/h+1/2)^k + \rho_b \]
\[ = (0.00000396 - 0.0000026989)(5/5+1/2)^2 + 0.0000026989 \]
\[ = 1.2611 \times 10^{-6} (2.25) + 0.0000026989 \]
\[ = 0.000005536375 \text{ Kg/mm}^3 \]

For \( k=2; \ z=-1 \)
\[ \rho(Z) = (\rho_t - \rho_b)(z/h+1/2)^k + \rho_b \]
\[ = (0.00000396 - 0.0000026989)(-1/5+1/2)^2 + 0.0000026989 \]
\[ = 1.2611 \times 10^{-6} (0.09) + 0.0000026989 \]
\[ = 0.000002812399 \text{ Kg/mm}^3 \]

For \( k=2; \ z=-2 \)
\[ \rho(Z) = (\rho_t - \rho_b)(z/h+1/2)^k + \rho_b \]
\[ = (0.00000396 - 0.0000026989)(-2/5+1/2)^2 + 0.0000026989 \]
\[ = 1.2611 \times 10^{-6} (0.01) + 0.0000026989 \]
\[ = 0.000002711511 \text{ Kg/mm}^3 \]

For \( k=2; \ z=-3 \)
\[ \rho(Z) = (\rho_t - \rho_b)(z/h+1/2)^k + \rho_b \]
\[ = (0.00000396 - 0.0000026989)(-3/5+1/2)^2 + 0.0000026989 \]
\[ = 1.2611 \times 10^{-6} (0.01) + 0.0000026989 \]
\[ = 0.000002711511 \text{ Kg/mm}^3 \]

For \( k=2; \ z=-4 \)
\[ \rho(Z) = (\rho_t - \rho_b)(z/h+1/2)^k + \rho_b \]
\[ = (0.00000396 - 0.0000026989)(-4/5+1/2)^2 + 0.0000026989 \]
\[ = 1.2611 \times 10^{-6} (0.09) + 0.0000026989 \]
\[ = 0.000002812399 \text{ Kg/mm}^3 \]

For \( k=2; \ z=-5 \)
\[ \rho(Z) = (\rho_t - \rho_b)(z/h+1/2)^k + \rho_b \]
\[ = (0.00000396 - 0.0000026989)(-5/10+1/2)^2 + 0.0000026989 \]
\[ = 1.2611 \times 10^{-6} (-0.25) + 0.0000026989 \]
\[ = 0.000002383625 \text{ Kg/mm}^3 \]

For \( K=2 \); Possions Ratio = 0.3
2.4. 2-D Drafting

![Figure 1](http://www.iaeme.com/images/f1.jpg)

**Figure 1** 2-D Drafting

3. STRUCTURAL ANALYSIS OF STIFFENED CYLINDRICAL SHELLS

3.1. Aluminum Alloy

Density : 2810 Kg/m³  
Young’s modulus : 71700 Mpa  
Poisson’s ratio : 0.33

![Figure 2](http://www.iaeme.com/images/f2.jpg)   ![Figure 3](http://www.iaeme.com/images/f3.jpg)

**Figure 2** Deformation  
**Figure 3** Strain

![Figure 4](http://www.iaeme.com/images/f4.jpg)

**Figure 4** Stress
3.2. Titanium Alloy

Density : 4700kg/m$^3$
Young's modulus : 110000 Mpa
Poisson’s ratio : 0.3

3.3. Modal Analysis

Figure 5 Deformation

Figure 6 Strain

Figure 7 Stress

Figure 8 Aluminium Alloy

Figure 9 Titanium Alloy
3.4. Buckling Analysis

Figure 10 Aluminium Alloy - Deformation

3.5. Shell Element for FGM

Structural Analysis

K=2

Figure 12 Total Deformation

Figure 13 Von-Mises Stress

3-Axis Micro CNC Machine

Figure 14 Von-Mises Strain
4. RESULTS TABLE

<table>
<thead>
<tr>
<th>Material</th>
<th>Deformation(mm)</th>
<th>Stress(N/mm$^2$)</th>
<th>Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>0.16327</td>
<td>141.39</td>
<td>0.0019926</td>
</tr>
<tr>
<td>Titanium</td>
<td>0.10328</td>
<td>143.77</td>
<td>0.0013076</td>
</tr>
<tr>
<td>K=2</td>
<td>0.1779</td>
<td>325.11</td>
<td>0.00072878</td>
</tr>
</tbody>
</table>

5. STRUCTURAL ANALYSIS

Graphs

**Figure 15**

**Figure 16**

**Figure 17**
6. MODAL ANALYSIS

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>MODE 1</th>
<th>MODE 2</th>
<th>MODE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deformation (mm)</td>
<td>Frequency (Hz)</td>
<td>Deformation (mm)</td>
</tr>
<tr>
<td>Al</td>
<td>57.1</td>
<td>1013.1</td>
<td>57.073</td>
</tr>
<tr>
<td>Ti</td>
<td>43.833</td>
<td>1000.9</td>
<td>43.811</td>
</tr>
<tr>
<td>K=2</td>
<td>35.139</td>
<td>3173.5</td>
<td>35.112</td>
</tr>
</tbody>
</table>

7. BUCKLING ANALYSIS

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deformation (mm)</td>
<td>Load Multiplier</td>
<td>Deformation (mm)</td>
</tr>
<tr>
<td>ALUMINUM</td>
<td>1.0033</td>
<td>3.5475</td>
<td>1.0045</td>
</tr>
<tr>
<td>TITANIUM</td>
<td>1.003</td>
<td>5.4844</td>
<td>1.0039</td>
</tr>
<tr>
<td>K=2</td>
<td>1</td>
<td>9.1742</td>
<td>1.00182</td>
</tr>
</tbody>
</table>
8. CONCLUSIONS

In this thesis finite element analysis is performed for a ring – stiffened cylindrical shell consisted of functionally graded material (FGM) subjected to hydrostatic pressure. Mathematical correlations are performed to determine the material properties of functionally graded material with metal Aluminum using Ceramic as interface zone for each layer up to 10 layers. FGM’s are considered for volume fractions of K=2 Structural analysis, Modal analysis and Buckling analysis is done to compare different materials Aluminum alloy, Titanium alloy and FGM.

By observing the structural analysis results, the displacements and stresses are more when FGM material is used when compared with that of aluminum alloy and titanium alloy. The stresses are less for Aluminum alloy.

By observing the modal analysis results, the frequencies are more for FGM material than aluminum alloy and titanium alloy. So vibrations are more for FGM materials than other materials. The deformations are less for FGM material than aluminum alloy and titanium alloy.

By observing buckling analysis results, the load multiplier is more for FGM material, that is the ring stiffened cylindrical shell fails by applying the load which is by increasing the present load times the load multiplier. So using FGM is better according to buckling analysis when compared with that of aluminum alloy and titanium alloy.

REFERENCES


Analysis of Ring Stiffened Cylindrical Shells Using Functionally Graded Material


