REDUCTION OF STRESS
CONCENTRATION BETWEEN PLATE CAM
AND ROLLER FOLLOWER BY ROUNding
THE EDGES

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ABSTRACT
This paper focuses on reduction of the stress concentration between a cam and follower by rounding the edges. A cam and its follower are designed for carrying out a simple harmonic motion. A CAD model of the flat edged cam is made with dimensions obtained from design calculations, followed by finite element analysis. Similarly, another CAD model of similar cam with rounded edges is made with the same dimensions, followed by finite element analysis. It can be found that the equivalent stress concentration is less at the rounded edges of the second model than the flat edges of the first model.

Key words: Cam, Cam and Follower, Stress concentration.

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1. INTRODUCTION
Cam and follower are commonly used machine components and a plate cam with a roller follower is most commonly used. Cam is a rotating piece which contacts with a follower to cause linear motion in the follower. It is often mounted on a shaft which is rotated by other machine components. The cam profile is designed according to requirement of linear motion of the cam follower. There are two types of cams, viz. Axial (cylindrical) cams and Radial (plate) cams. Motions are generally classified as Uniform acceleration & retardation, Uniform motion, Simple harmonic motion and Cycloidal motion. Different combinations of these motions can be used in a single cam.

Stress concentration can be reduced in many ways, such as rounding corners, providing holes near cavities, etc. [1]

In this paper, a cam with roller follower is designed. This cam provides simple harmonic motion to the follower. By using the dimensions calculated, a two CAD models are designed, one with flat rolling surface on cam and flat roller follower,
while other with curved edges of the cam and roller follower. Finite element analysis is done to find the stress concentration between the cam and roller. It is then found that the stress concentration is reduced when the edges are rounded.

2. DESIGN OF CAM AND FOLLOWER
For this study, we will consider a rotary cam with central translatory roller follower. It contains the following motions

- Forward stroke of 20mm in 100° of cam rotation with simple harmonic motion.
- Dwell for 40° of cam rotation.
- Return stroke of 120° of cam rotation with simple harmonic motion.
- Dwell for 100° of cam rotation.

Mass of the follower (m) = 1 kilogram.
Cam shaft speed = 500rpm
External force during forward stroke = 500N.
External force during return stroke = 200N.

2.1. Motion analysis of the follower

2.1.1. Displacement analysis
For simple harmonic motion for forward and return stroke, displacement at an angle θ is given by

\[ y = \frac{h}{2} \left(1 - \cos \frac{\pi \theta}{\beta}\right) \]  

Where, h = stroke length
\[ \beta = \text{angle turned in the stroke} \]

2.1.2. Velocity Analysis
For simple harmonic motion for forward and return stroke, velocity at an angle θ is given by

\[ v = \frac{h \pi \omega}{2 \beta} \sin \frac{\pi \theta}{\beta} \]  

\[ \omega = 2\pi \times \text{rpm} \times \frac{\rho}{30} \]

![Figure 1](http://www.iaeme.com/IJMET/index.asp) Displacement, velocity and acceleration graphs of the motion
From Figure 1 it is observed that at points 1, 3, 4, 6, velocity is zero. For forward stroke,
At \( \beta_1 \), \( v = v_{\text{maximum}} \)
\( v_{\text{maximum}} = 0.94248 \text{ m/s} \)
For return stroke,
At \( \beta_2 \), \( v = v_{\text{maximum}} \)
\( v_{\text{maximum}} = 0.78892 \text{ m/s} \)

2.1.3. Acceleration analysis
For simple harmonic motion for forward and return stroke, acceleration at an angle \( \theta \) is given by
\[
\alpha = \frac{h}{2} \left( \frac{\pi \omega}{\beta} \right)^2 \cos \left( \frac{\pi \theta}{\beta} \right) \quad [2]
\]
From Figure 1 it is observed that at points 3 and 5 acceleration is zero. For forward stroke,
At point 1, = 0 , \( a = a_{\text{maximum}} \)
\( a_{\text{maximum}} = 88.826 \text{ m/s}^2 \)
At point 2, \( \theta = 0 \), \( a = a_{\text{maximum}} \)
\( a_{\text{maximum}} = -88.826 \text{ m/s}^2 \)
For return stroke,
At point 4, = 0 , \( a = a_{\text{maximum}} \)
\( a_{\text{maximum}} = 61.6853 \text{ m/s}^2 \)
At point 6, \( \theta = 0 \), \( a = a_{\text{maximum}} \)
\( a_{\text{maximum}} = -61.6853 \text{ m/s}^2 \)

2.2. Calculation of \( R_p, r_p, \rho_{k\text{min}}, \rho_{c\text{min}} \)
For forward stroke,
\[
\tan \alpha_{\text{maximum}} = \frac{dy}{d\theta} \left( \rho_{k\text{min}} + \rho_{c\text{min}} \right) \quad [2]
\]
\[
\frac{dy}{d\theta} = \frac{v}{w}
\]
\[
\rho_{k\text{min}} = \frac{\pi}{2} \left( 1 - \frac{\cos \frac{\pi \beta_1}{2}}{\pi \beta_1} \right) \quad [2]
\]
\( = 0.01 \text{ m} \)
Let the maximum pressure angle be \( 25^\circ \)
\[
\tan 25^\circ = \frac{v/w}{R_p + 0.01} \quad [2]
\]
\( \therefore R_p = 22 \text{ mm} \)
Reduction of Stress Concentration Between Plate Cam and Roller Follower by Rounding The Edges

Selecting $R_p$ with the higher value to avoid interference and undercutting of cam

$\therefore R_p = 29\text{mm}$

Roller radius $= R_r = R_p/5 = 5.8\text{mm}$

$\therefore d = 11.6\text{mm}$

Base circle radius,

$r_p = R_p - R_r = 23.2\text{mm}$

For Simple harmonic motion,

\[
\tan \varphi_{max} = \frac{\pi}{p} \cot \left( \frac{n\delta_p}{p} \right)
\]

Pitch point angle $\theta_p = 41.93^o$

$\therefore y_{\theta_p} = 7.4913\text{m}$

Pitch circle radius $= R_p + y_{\theta_p}$ \[2\]

$= 36.49\text{mm}$

\[
\rho_{kmin} = \frac{(R_p + y)^2 + \left( \frac{1}{w} \frac{dy}{dt} \right)^2}{(R_p + y)^2 + 2 \left( \frac{1}{w} \frac{dy}{dt} \right)^2 - (R_p + y)^2 \left( \frac{1}{w} \frac{d^2y}{dt^2} \right)^2}
\]

$\rho_{kmin} = 29.496\text{mm}$

Radius of Cam profile

$\rho_{cmin} = \rho_{kmin} - R_r$

$\rho_{cmin} = 23.696\text{mm}$

2.3. Force analysis

Resultant force acting on cam is given by

$F_R = mg + m\ddot{x} + F_{external}$ \[3\]

Therefore, resultant forces at points 1, 2, 3, 4, 5, 6 are,

<table>
<thead>
<tr>
<th>Points</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>598.64</td>
</tr>
<tr>
<td>2</td>
<td>509.81</td>
</tr>
<tr>
<td>3</td>
<td>420.98</td>
</tr>
<tr>
<td>4</td>
<td>48.12</td>
</tr>
<tr>
<td>5</td>
<td>109.81</td>
</tr>
<tr>
<td>6</td>
<td>171.49</td>
</tr>
</tbody>
</table>
From Table (1)
At point 1,
\[ F_{\text{maximum}} = 598.64 \text{ N} \]
Also, at point 4,
\[ F_{\text{minimum}} = 48.12 \text{ N} \]
As minimum force \( F_{\text{minimum}} > 30 \text{ N} \), Spring is not required. \[ \ldots \] [2]
Now,
Normal force acting on cam surface is given by,
\[ P_n = \frac{F_{\text{maximum}}}{\cos(\alpha_{\text{maximum}})} \ldots \] [3]
\[ = 660.52 \text{ N} \]
Width of the cam can be determined by the following relation
\[ \sigma_c = 0.04 \sqrt{\frac{P_n(E_1E_2)}{b(E_1+E_2)} \left( \frac{1}{R_{\text{minimum}}} + \frac{1}{R_n} \right)} \ldots \] [2]
Consider roller material as hardened steel and cam material as C20
\[ \therefore \sigma = 5000 \frac{\text{kgf}}{\text{cm}^2} = 500 \text{ N/mm}^2 \ldots \] [2]
For Steel \( E_1 = E_2 = 2.1\times10^5 \text{ N/mm}^2 \)
\[ \therefore b = 20.778 \text{ mm} \]
Rounding off to next even value.
\[ b \approx 22 \text{ mm} \]

3. ANALYSIS

3.1. CAD model
A CAD model was constructed in Solidworks 2015[4] with the dimensions obtained from the above calculations.
Considering the point of contact to be one with maximum resultant force, i.e. point 1. (From Figure 1)

3.1.1. Cam and follower without rounded edges

Figure 2 Cam and follower without rounded edges
3.1.2. Cam and follower with rounded edges
Fillet of radius 2.5mm was provided to the cam and roller edge.

![Figure 3 Cam and follower with rounded edges](image)

3.2. Meshing
Meshing was carried out in Ansys Workbench [5]

3.2.1. Cam and follower without rounded edges

<table>
<thead>
<tr>
<th>Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes</td>
<td>111128</td>
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<tr>
<td>Elements</td>
<td>27612</td>
</tr>
</tbody>
</table>

![Figure 4 Meshing of Cam and follower without rounded edges](image)

3.1.2. Cam and follower with rounded edges

<table>
<thead>
<tr>
<th>Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes</td>
<td>139634</td>
</tr>
<tr>
<td>Elements</td>
<td>33853</td>
</tr>
</tbody>
</table>
3.3. Boundary Conditions
A load of 598.64 N is applied on the follower as it is the maximum force at point 1 (From figure 1).

3.4. Results
Equivalent (Von misses) stress between cam and roller

3.4.1. Cam and follower without rounded edges

The maximum equivalent (Von misses) stress is found to be \(4.1684 \times 10^8\) Pa in the cam and follower without rounded edges.
3.4.2. Cam and follower with rounded edges

The maximum equivalent (Von misses) stress is found to be $1.8839 \times 10^8$ Pa in the cam and follower with rounded edges.

Therefore, it is found that stress is reduced between cam and roller when the edges are rounded.

4. CONCLUSION

The results clearly show that rounding off the edges of the cam and roller of the follower reduces the stress between them. This method may not be the only method of reducing stress concentration between cam and roller follower. Future work can be carried out on other ways to reduce stress concentration between them. Full optimization is beyond the scope of this paper.
5. GLOSSARY
The terms and abbreviations used in the calculations are as follows,

<table>
<thead>
<tr>
<th>Term</th>
<th>Meaning</th>
<th>Unit</th>
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<tbody>
<tr>
<td>h</td>
<td>Length of Stroke</td>
<td>m</td>
</tr>
<tr>
<td>m</td>
<td>Mass of follower</td>
<td>kg</td>
</tr>
<tr>
<td>N</td>
<td>Number of revolutions</td>
<td>RPM</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Angle of turn</td>
<td>Degrees</td>
</tr>
<tr>
<td>y</td>
<td>Distance travelled</td>
<td>m</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Angle of stroke</td>
<td>Degrees</td>
</tr>
<tr>
<td>v</td>
<td>Velocity of stroke</td>
<td>m/s</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Angular velocity</td>
<td>Rad/s</td>
</tr>
<tr>
<td>a</td>
<td>Acceleration of stroke</td>
<td>m/s^2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Pressure angle</td>
<td>Degrees</td>
</tr>
<tr>
<td>R_p</td>
<td>Prime circle radius</td>
<td>m</td>
</tr>
<tr>
<td>R_r</td>
<td>Roller radius</td>
<td>m</td>
</tr>
<tr>
<td>( \theta_p )</td>
<td>Pitch Point angle</td>
<td>Degrees</td>
</tr>
<tr>
<td>y_{\theta_p}</td>
<td>Pitch point follower displacement</td>
<td>m</td>
</tr>
<tr>
<td>R_s</td>
<td>Pitch circle radius</td>
<td>m</td>
</tr>
<tr>
<td>( \rho_{c_{_{minimum}}} )</td>
<td>Radius</td>
<td>m</td>
</tr>
<tr>
<td>F_R</td>
<td>Resultant force</td>
<td>N</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to</td>
<td>m/s^2</td>
</tr>
<tr>
<td>P_N</td>
<td>Normal force acting on cam surface</td>
<td>N</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus</td>
<td>N/mm^2</td>
</tr>
<tr>
<td>b</td>
<td>Width of cam</td>
<td>m</td>
</tr>
</tbody>
</table>

REFERENCES