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JANOWSKI-SAKAGUCHI TYPE FUNCTIONS ASSOCIATED WITH CONIC REGIONS

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ABSTRACT

The purpose of the present paper is to introduce and investigate some new subclasses of Sakaguchi type functions defined by using the concept of Janowski functions in conic regions. Various interesting properties such as sufficient criteria, coefficient estimates and distortion results are investigated for these classes.

Keywords: Analytic functions, Conic domains, Janowski functions, Sakaguchi type functions, 2010AMS Subject Classification: 30C45, 30C50

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1. INTRODUCTION

Let A be the class of functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the open unit disk $U = \{z : z \in C \text{ and } |z| < 1\}$ and S represents the class of all functions in A which are univalent in U. If f and g are analytic functions in U then we say that f is subordinate to g denoted by $f(z) \prec g(z)$, if there exists an analytic function w in U with $|w(z)| < |z|$ such that $f(z) = g(w(z))$. Furthermore if the function g is univalent in U then we have the following equivalence:

$$f \prec g \Leftrightarrow f(0) = g(0) \text{ and } f(U) \subset g(U).$$

Let P denote the class of analytic functions of the form

$$p_n(z) = 1 + \sum_{n=1}^{\infty} P_n z^n,$$

Defined on unit disk U and satisfying $p(0) = 1$ and $\Re(p(z)) > 0,$ for all $z \in U.$

Let $P[A,B]$ denote the class of analytic functions p defined on U with the representation

$$p(z) = \frac{1 + Aw(z)}{1 + Bw(z)},$$

Where $w(0) = 0,\ |w(z)| < 1.$ We say that $p \in P[A,B]$ if and only if $p(z) \prec \frac{1 + Az}{1 + Bz},$ where $-1 \le B < A \le 1.$

Geometrically, a function $p(z) \in P[A,B]$ maps the unit disk U onto the disk defined by the domain

$$\Omega[A,B] = \left\{ w : \left| w - \frac{1 - AB}{1 - B^2} \right| < \frac{A - B}{1 - B^2} \right\}.$$

The relation between the classes P and $P[A,B]$ is expressed as follows:

$$p(z) \in P \Leftrightarrow \frac{(A+1)\,p(z) - (A-1)}{(B+1)\,p(z) - (B-1)} \in P[A,B].$$

This class was introduced by Janowski [1] and then studied by several authors [2,3,4,5,6,7,8].

Kanas and Wisniowska [10, 11] introduced and studied by the class $k-UCV$ of $k-$ uniformly convex functions and the corresponding class $k-ST$ of $k-$ starlike functions. These classes were defined subject to the conic region $\Omega_k,\ k \ge o$ given by

$$\Omega_k = \left\{ u + iv : u > k\sqrt{(u-1)^2 + v^2} \right\} \tag{2}$$

This domain represents the right half plane for $k = 0,$ hyperbola for $0 < k < 1,$ a parabola for $k = 1$ and ellipse for $k > 1.$

They also extend this domain to $\Omega_{k,\sigma}$ defined by

$$\Omega_{k,\sigma} = (1-\sigma)\Omega_k + \sigma(0 \le \sigma < 1).$$

The function $p_{k,\sigma}$ with $p_{k,\sigma}(0) = 1,\ p'_{k,\sigma} > 0$ plays the role of external for these conic domains $\Omega_{k,\sigma}$ is given by

$$p_{k,\sigma}(z)=\begin{cases}\dfrac{1+(1-2\sigma)z}{1-z}, \; k=0\\ 1+\dfrac{2(1-\sigma)}{\pi^2}\left(\log\dfrac{1+\sqrt{z}}{1-\sqrt{z}}\right)^2, \; k=1\\ 1+\dfrac{2(1-\sigma)}{1-k^2}\sinh^2\left[\left(\dfrac{2}{\pi}\arccos k\right)\arctan h\sqrt{z}\right], \; 0<k<1\\ 1+\dfrac{(1-\sigma)}{k^2-1}\sin\left[\dfrac{\pi}{2R(t)}\int_0^{\frac{u(z)}{\sqrt{t}}}\dfrac{1}{\sqrt{1-x^2}\sqrt{1-(tx)^2}}dx\right]+\dfrac{1}{k^2-1}, \; k>1\end{cases} \tag{3}$$

Where $u(z)=\dfrac{z-\sqrt{t}}{1-\sqrt{tx}}$, $t\in(0,1)$, $z\in U$ *and* t is chosen such that $k=\cosh\left(\dfrac{\pi R'(t)}{4R(t)}\right)$, with $R(t)$ is the Legendre's complete elliptic integral of the first kind and $R'(t)$ is complementary integral $R(t)$.

Let $P_{pk,\sigma}$ denote the class of all functions p which are analytic in U with $p(0)=1$ and $p(z)\prec p_{k,\sigma}(z)$ for $z\in U$ Clearly, it can be seen that $P_{k,\sigma}(z)\subset P$.

Definition 1.1. (See [12]) A function p is said to be in the class $k-P[A,B]$, if and only if

$$p(z)\prec\frac{(A+1)p_k(z)-(A-1)}{(B+1)p_k(z)-(B-1)}, \quad k\geq 0 \tag{4}$$

Where $p_k(z)$ is defined by (3) and $-1\leq B<A\leq 1$.

Geometrically, the function $p\in k-P[A,B]$ takes all values form the domain $\Omega_k[A,B]$, $-1\leq B<A\leq 1$, $k\geq 0$ which is defined as

$$\Omega_k[A,B]=\left\{w:\Re\left(\frac{(B-1)w(z)-(A-1)}{(B+1)w(z)-(A+1)}\right)>k\left|\frac{(B-1)w(z)-(A-1)}{(B+1)w(z)-(A+1)}-1\right|\right\} \tag{5}$$

Or equivalently

$$\Omega_k[A,B]=\Big\{u+iv:\left[(B^2-1)(u^2+v^2)-2(AB-1)u+(A^2-1)\right]^2$$
$$>k^2\left[\left(-2(B+1)(u^2+v^2)+2(A+B+2)u-2(A+1)\right)^2+4(A-B)^2v^2\right]\Big\}.$$

The domain $\Omega_k[A,B]$ retains the conic domain Ω_k inside the circular region defined by $\Omega[A,B]$ the impact of $\Omega[A,B]$ on the conic domain Ω_k changes the original shape of the conic regions. The ends of hyperbola and parabola get closer to each other but never meet anywhere and the ellipse and the ellipse gets the oval shape. When $A\to 1, B\to 1,$ the radius of the circular disk defined by $\Omega[A,B]$ tends to infinity, consequently, the arms of hyperbola and parabola expand and the oval turns into ellipse. We see that $\Omega_k[1,-1]=\Omega_k$, the conic domain defined by Kanas and Wisniowska [10, 11].

Motivated essentially by the recent paper of Noor and Malik [12], we define some classes of analytic functions associated with conic domains as follows:

Definition 1.2. A function p is said to be in the class $k-P[A,B,\sigma]$, if and only if

$$p(z)\prec\frac{(A+1)p_{k,\sigma}(z)-(A-1)}{(B+1)p_{k,\sigma}(z)-(B-1)},\quad k\geq 0$$

Where $p_{k,\sigma}(z)$ is defined by $(3) 0\leq\sigma<1$ and $-1\leq B<A\leq 1$.

Definition 1.3. A function $f\in U$ is said to be in the class $k-US[A,B,\sigma,s,t]$, if and only if

$$\frac{(s-t)zf'(z)}{f(sz)-f(tz)}\in k-P[A,B,\sigma],\ z\in U,\ k\geq 0\ s,t\in C\ with\ s\neq t.$$

Definition 1.4. A function $f\in A$ is said to be in the class $k-UC[A,B,\sigma,s,t]$, if and only if

$$\frac{(s-t)(zf'(z))'}{sf'(sz)-tf'(tz)}\in k-P[A,B,\sigma],\ z\in U,\ k\geq 0\ s,t\in C\ with\ s\neq t.$$

It can easily be seen that

$$f(z)\in k-UC[A,B,s,t]\Leftrightarrow zf'(z)\in k-US[A,B,s,t]. \qquad (6)$$

Special cases we get the classes defined by Janowwski [1], Khalida Inyat Noor and Sarfraz Nawaz Malik [12], Kanas [10] and Wisniowska [11], Shams [7].

The purpose of this paper is to define new a new class of functions by using the concepts of Janowski functions in conic regions. Certain interesting coefficient inequalities are also discussed.

2. MAIN RESULTS

To prove our main results, we need the following two lemmas.

Lemma 2.1. Let $p(z)=1+\sum_{n=1}^{\infty}p_nz^n\prec F(z)=1+\sum_{n=1}^{\infty}d_nz^n z\in U$. If F is univalent in U and $F(U)$ is convex, then $|p_n|\leq|d_1|, n\geq 1$.

Lemma 2.2. Let $p(z)=1+\sum_{n=1}^{\infty}c_nz^n\prec k-P[A,B,\sigma]$. Then $|c_n|\leq|\delta_{AB}|$

Where

$$|\delta_{AB}|=\frac{(A-B)|\delta_{k,\sigma}|}{2} \qquad (7)$$

and

$$\delta_{k,\sigma} = \begin{cases} \dfrac{8(1-\sigma)(\arccos k)^2}{\pi^2(1-k^2)}, 0 \le k < 1 \\ \dfrac{8(1-\sigma)}{\pi^2}, k = 1 \\ \dfrac{\pi^2(1-\sigma)}{4(k^2-1)\sqrt{t(1+t)R^2(t)}}, k > 1 \end{cases} \tag{8}$$

Theorem 2.3. Let $f \in k-US[A,B,\sigma,s,t]$. Then $|a_2| \le \frac{\delta_{AB}}{2-u_2}$, and for $n \ge 3$

$$|a_n| \le \prod_{i=1}^{n-1} \frac{|\delta_{k,\sigma}(A-B) - 2(i-u_i(s,t))B|}{2(i+1-u_n(s,t))}$$

Where δ_{AB} is defined in (7) and

$$u_n(s,t) = \sum_{i=0}^{\infty} s^{n-i}t^{n-i} s,t \in C \text{ with } s \ne t. \tag{9}$$

Proof. Consider

$$\frac{(s-t)zf'(z)}{f(sz)-f(tz)} = p(z) \tag{10}$$

Where

$$p(z) \prec \frac{(A+1)p_k(z)-(A-1)}{(B+1)p_k(z)-(B-1)}$$

$$= \left[(A+1)p_k(z)-(A-1)\right]\left[(B+1)p_k(z)-(B-1)\right]^{-1}$$

$$= \frac{A-1}{B-1}\left[1-\frac{A+1}{A-1}p_k(z)\right]\left[1+\sum_{n=1}^{\infty}\left(\frac{B+1}{B-1}p_k(z)\right)^n\right]$$

$$= \frac{A-1}{B-1} + \left(\frac{(A-1)(B+1)}{(B-1)^2} - \frac{A+1}{B-1}\right)p_k(z) + \left(\frac{(A-1)(B+1)^2}{(B-1)^3} - \frac{(A+1)(B+1)}{(B-1)^2}\right)(p_k(z))^2$$
$$+ \left(\frac{(A-1)(B+1)^3}{(B-1)^4} - \frac{(A+1)(B+1)^2}{(B-1)^3}\right)(p_k(z))^3 + \dots.$$

If $p_k(k) = 1 + \delta_k z + \dots$ then we have after suitable simplification

$$p(z) \prec \sum_{n=1}^{\infty} \frac{-2(B+1)^{n-1}}{(B-1)^n} + \left\{\sum_{n=1}^{\infty} \frac{2n(A-B)(B+1)^{n-1}}{(B-1)^{n+1}}\right\}\delta_k z + \dots.$$

Now we see that series $\sum_{n=1}^{\infty} \frac{-2(B+1)^{n-1}}{(B-1)^n}$ and $\sum_{n=1}^{\infty} \frac{2n(A-B)(B+1)^{n-1}}{(B-1)^{n+1}}$ are convergent and converge to 1 and $\frac{A-B}{2}$ respectively.

Therefore,

$$p(z) \prec 1+\frac{1}{2}(A-B)\delta_{k,\sigma} z+....$$

Now if $p(z)=1+\sum_{n=1}^{\infty} c_n z^n$, then by Lemma 2.2 =, we get $|c_n| \leq \frac{1}{2}(A-B)\delta_{k,\sigma}, n \geq 1$ Now from, we have

$$z+\sum_{n=2}^{\infty} n a_n z^n = \left[z+\sum_{n=2}^{\infty} u_n(s,t) a_n z^n \right]\left[1+\sum_{n=1}^{\infty} c_n z^n \right]$$

Equating coefficients of z^n on both sides, we have

$$(n-u_n(s,t))a_n = \sum_{i=1}^{n-1} u_{n-i}(s,t) a_{n-i} c_i .$$

This implies that

$$|a_n| \leq \frac{\delta_{AB}}{|n-u_n(s,t)|} \sum_{i=1}^{n-1} |u_{n-i}(s,t)||a_{n-i}||c_i| \tag{11}$$

We get

$$|a_n| \leq \frac{|\delta_{k,\sigma}|(A-B)}{2|n-u_n(s,t)|} \sum_{i=1}^{n-1} |u_i(s,t)||a_i|.$$

Now we prove that

$$\frac{|\delta_{k,\sigma}|(A-B)}{2|n-u_n(s,t)|} \sum_{i=0}^{n-1} |u_i(s,t)||a_n| \leq \prod_{i=0}^{n-1} \frac{|\delta_{k,\sigma}|(A-B)-2|i-u_i(s,t)|B}{2|i+1-u_{i+1}(s,t)|} \tag{12}$$

For $n=2,$ we have

$$|a_2| \leq \frac{|\delta_{k,\sigma}|(A-B)}{2|2-u_2(s,t)|}$$

We have

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For $n=3,$ we have

$$|a_3| \le \frac{|\delta_{k,\sigma}|(A-B)}{2|3-u_3(s,t)|}\left[1+\frac{|\delta_{k,\sigma}|(A-B)}{2|2-u_2(s,t)|}\right].$$

Let the hypothesis be true for $n = m.$ we have

$$|a_m| \le \prod_{i=1}^{m-1} \frac{|\delta_{k,\sigma}|(A-B)-2|i-u_i(s,t)|B}{2|i+1-u_{i+1}(s,t)|}$$

$$\le \prod_{i=1}^{m-1} \frac{|\delta_{k,\sigma}|(A-B)+2|i-u_i(s,t)|}{2|i+1-u_{i+1}(s,t)|}$$

By the induction hypothesis, we have

$$\frac{|\delta_{k,\sigma}|(A-B)}{2|m-u_m(s,t)|}\sum_{i=1}^{m-1} u_i(s,t)|a_i| \le \prod_{i=1}^{m-1} \frac{|\delta_{k,\sigma}|(A-B)+2|i-u_i(s,t)|}{2|i+1-u_{i+1}(s,t)|}$$

Multiplying both sides by $\frac{|\delta_{k,\sigma}|(A-B)+2|m-u_m(s,t)|}{2|m+1-u_{m+1}(s,t)|}$, we have

$$\prod_{i=1}^{m} \frac{|\delta_{k,\sigma}|(A-B)+2|i-u_i(s,t)|}{2|i+1-u_{i+1}(s,t)|} \ge \frac{|\delta_{k,\sigma}|(A-B)}{2|m-u_m(s,t)|}.\frac{|\delta_{k,\sigma}|(A-B)+2|m-u_m(s,t)|}{2|m+1-u_{i+1}(s,t)|}\sum_{i=1}^{m-1}|u_i(s,t)||a_i|$$

$$= \frac{|\delta_{k,\sigma}|(A-B)}{2|m+1-u_t(m+1)|}\left[\frac{|\delta_{k,\sigma}|(A-B)}{2|m-u_m(s,t)|}\sum_{i=1}^{m-1} u_i(s,t)|a_i| + \sum_{i=1}^{m-1} u_i(s,t)|a_i|\right]$$

$$\ge \frac{|\delta_{k,\sigma}|(A-B)}{2|m+1-u_{m+1}(m+1)|}\left[|u_m(s,t)||a_m| + \sum_{i=1}^{m-1}|u_{m-1}(s,t)||a_i|\right]$$

$$= \frac{|\delta_{k,\sigma}|(A-B)}{2|m+1-u_{m+1}(s,t)|}\sum_{i=1}^{m}|u_{m-1}(s,t)|a_i||$$

That is

$$\frac{|\delta_{k,\sigma}|(A-B)}{2|m+1-u_{m+1}(s,t)|}\sum_{i=1}^{m}|u_i(s,t)||a_i| \le \prod_{i=1}^{m} \frac{|\delta_{k,\sigma}|(A-B)+2|i-u_i(s,t)|}{2|i+1-u_{i+1}(s,t)|}$$

Which shows that inequality is true for $n = m+1.$ Hence the result.

For $t = 0, s = 1$ *and* $\sigma = 0,$ we get the following result obtained in [12].

Corollary 2.4. Let $f \in k - ST[A,B].$ Then

$$|a_n| \le \prod_{i=0}^{n-2} \frac{|\delta_{k,\sigma}(A-B)-2iB|}{2(i+1)}(n \ge 2).$$

Furthermore, by taking $A = 1$ and $B = -1$ in the last Corollary 2.4, we obtain the following result for the class $k = ST[A,B],$ which was introduced by Kanas and Wisniowska [10, 11].

Corollary 2.5. Let $f \in k-ST\left[A,B\right]$. Then

$$\left|a_n\right| \le \prod_{i=0}^{n-2} \frac{\left|\delta_{k,\sigma}+i\right|}{(i+1)}(n \ge 2).$$

By putting $A=1-2\beta,\ 0 \le \beta < 1,\ B=-1,\ t=0\ s=1\ and\ \sigma=0$ in Theorem 9, we obtain the coefficient bounds for $SD\left[k,\beta\right]$ defined by Shams et al. [7].

Corollary 2.6. Let $f \in k-SD\left[k,\beta\right]$. Then

$$\left|a_n\right| \le \prod_{i=0}^{n-2} \frac{\left|\delta_{k,\sigma}(1-\beta)+i\right|}{(i+1)}(n \ge 2).$$

The above result is obtained by Owa et al. [14].

By setting $t=0,\ s=1,\ \sigma=0,\ k=0\ and\ \delta_{k,\sigma}=2$ in Theorem 2.3, we get the following coefficient bounds which we proved by Janowski [1].

Corollary 2.7. Let $f \in S^*\left[A,B\right]$. Then

$$\left|a_n\right| \le \prod_{i=0}^{n-2} \frac{\left|(A-B)-iB\right|}{(i+1)}(n \ge 2).$$

Further by setting $A=1-2\beta$ with $0 \le \beta < 1,\ B=-1,$ we obtain the coefficient estimates for the class $S^*(\beta)$ introduced by Robertson [16].

Theorem 2.8. Let $f \in k-UC\left[A,B,\sigma,s,t\right]$. Then $\left|a_2\right| \le \dfrac{\delta_{AB}}{2-u_2}$ and for $n \ge 3$

$$\left|a_n\right| \le \prod_{i=1}^{n-1} \frac{\left|\delta_{k,\sigma}(A-B)-2\left(i-u_i(s,t)\right)B\right|}{2\left(i+1-u_n(s,t)\right)}$$

Where δ_{AB} is defined in (7) and

$$u_n(s,t)=\sum_{i=0}^{\infty} s^{n-i}t^{n-i} s,t \in C\ with\ s \ne t \tag{13}$$

Proof. By virtue of Theorem 2.3, and the relationship (6) we get the required result.

Theorem 2.9. A function $f \in A$ and of the form (6) is in the class $k-US\left[A,B,\sigma,s,t\right]$, if it satisfies the condition

$$\sum_{n=2}^{\infty}\left\{\frac{(s-t)(k+1)}{(1-\sigma)}\left|n-u_n(s,t)\right|+\left|n(B+1)-u_n(s,t)(A+1)\right|\right\}\left|a_n\right| < \left|B-A\right|, \tag{14}$$

Where $-1 \le B < A \le 1,\ k \ge 0\ and\ u_n(s,t)=\sum_{j=0}^{\infty} s^{n-j}t^{n-j}\ s,t \in C\ with\ s \ne t.$

Proof. Assuming that (14) holds, then show that

$$k\left|\frac{(B-1)\frac{(s-t)zf'(z)}{f(sz)-f(tz)}-(A-1)}{(B+1)\frac{(s-t)zf'(z)}{f(sz)-f(tz)}-(A+1)}-1\right|-\Re\left[\frac{(B-1)\frac{(s-t)zf'(z)}{f(sz)-f(tz)}-(A-1)}{(B+1)\frac{(s-t)zf'(z)}{f(sz)-f(tz)}-(A+1)}-1\right]<1$$

We get

$$k\left|\frac{(B-1)\frac{(s-t)zf'(z)}{f(sz)-f(tz)}-(A-1)}{(B+1)\frac{(s-t)zf'(z)}{f(sz)-f(tz)}-(A+1)}-1\right|-\Re\left[\frac{(B-1)\frac{(s-t)zf'(z)}{f(sz)-f(tz)}-(A-1)}{(B+1)\frac{(s-t)zf'(z)}{f(sz)-f(tz)}-(A+1)}-1\right]$$

$$\leq(k+1)\left|\frac{(B-1)(s-t)zf'(z)-(A-1)f(sz)-f(tz)}{(B+1)(s-t)zf'(z)-(A+1)f(sz)-f(tz)}-1\right|$$

$$=(s-t)(k+1)\left|\frac{f(sz)-f(tz)-zf'(z)}{(B+1)zf'(z)-(A+1)f(sz)-f(tz)}\right|$$

$$=(s-t)(k+1)\left|\frac{\sum_{n=2}^{\infty}(u_n(s,t)-n)a_nz^n}{(B-A)z+\sum_{n=2}^{\infty}\left[n(B+1)-u_n(s,t)(A+1)\right]a_nz^n}\right|$$

$$\leq(s-t)(k+1)\left|\frac{\sum_{n=2}^{\infty}|u_n(s,t)-n||a_n|z^n}{|B-A|-\sum_{n=2}^{\infty}|n(B+1)-u_n(s,t)(A+1)||a_n|}\right|$$

The last expression is bounded above by $1-\sigma,$ then

$$\sum_{n=2}^{\infty}\left\{\frac{(s-t)(k+1)}{1-\sigma}|n-u_n(s,t)|+|n(B+1)-u_n(s,t)(A+1)|\right\}|a_n|<|B-A|,$$

This completes the proof.

When $s=1$ and $t=0$ we have the following known result, proved by Khalida Inayat Noor and Sarfraz Nawaz Malik in [3.5].

Corollary 2.10. A function $f\in A$ and form (6) in the class $k-ST[A,B],$ if it satisfies the condition

$$\sum_{n=2}^{\infty}\left\{2(k+1)(n-1)+|n(B+1)-(A-1)|\right\}|a_n|<|B-A|, \tag{15}$$

Where $-1\leq B<A\leq 1 \; and \; k\geq 0.$

For $t=0,\ s=1\ A=1\ and\ B=-1,$ we have following result proved by Kanas and Wisniowska [10, 11].

Corollary 2.11. A function $f\in A$ and form (6) in the class $k-ST$ if it satisfies the condition

$$\sum_{n=2}^{\infty}\{n+k(n-1)\}|a_n|<1, k\geq 0. \tag{16}$$

For $t=0,\ s=1\ A=1-2\alpha\ and\ B=-1\ with\ 0\leq\alpha<1,$ we arrive at Shams et. result in [7].

Corollary 2.12. A function $f\in A$ and form (6) in the class $SD(k,\alpha)$, if it satisfies the condition

$$\sum_{n=2}^{\infty}\{n(k+1)-(k+\alpha)\}|a_n|<1-\alpha, \tag{17}$$

Where $0\leq\alpha<1\ and\ k\geq 0.$

Also for $t=0,\ s=1\ A=1-2\alpha\ and\ B=-1,\ k=0\ with\ 0\leq\alpha<1,$ then we get the well-known Silverman's results [15].

Corollary 2.13. A function $f\in A$ and form (6) in the class $S^*(\alpha)$, if it satisfies the condition

$$\sum_{n=2}^{\infty}\{(n-\alpha)\}|a_n|<1-\alpha, \tag{18}$$

Where $0\leq\alpha<1.$

Theorem 2.14. A function $f\in A$ and of the form (6) is in the class $k-UCV[A,B,\sigma,s,t]$, if it satisfies the condition

$$\sum_{n=2}^{\infty}n\left\{\frac{(s-t)(k+1)}{(1-\sigma)}|n-u_n(s,t)|+|n(B+1)-u_n(s,t)(A+1)|\right\}|a_n|<|B-A|$$

Where $-1\leq B<A\leq 1,\ k\geq 0\ and\ u_n(s,t)=\sum_{j=0}^{\infty}s^{n-j}t^{n-j}\ s,t\in C.$

Proof. The proof follow directly by Theorem 2.9 and (6).

3. CONCLUSION

We have established Coefficient inequalities for Janowski-Sakaguchi type functions defined through Conic regions. The applications of the above inequalities for subclass of functions defined by convolution with a class of analytic functions are also defined.

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