METHODS OF ASSESSING THE OPTIMAL PROCEDURE FOR FORMING MUTUAL INSURANCE FUNDS

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ABSTRACT

The article reviews methods of assessing the efficiency of alternative strategies of capital accumulation by mutual insurance societies (MIS) in the context of random flows of insurance payments, based on the criteria expressing costs of the insurant on an initial and current payments and probabilities of a society default. Possible mechanisms for improving the MIS financial stability related to its subsidization and concessional lending by the state, region, and industry, i.e. a Center, are discussed. The options of setting tasks for optimal management of the process of MIS accumulation and their numerical solutions obtained on the basis of simulation modeling methods are provided.

It is demonstrated that optimization of the fund formation strategies can significantly reduce the cost of participation in the MIS due to the reduced amount of current contributions. The availability of an external source of MIS funding allows to ensure an even greater reduction in the cost of insurance, which, in case of realization of the optimal procedure for interaction, is in direct proportion to the difference in the rates of return on the capital of the MIS participants and the Center.

Keywords: insurance protection, mutual insurance, insurance premium, stability of insurance institutions, optimal management, mixed financing, simulation modeling.


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1. INTRODUCTION

The mutual insurance societies (MIS), the insurance fund of which is accumulated from their insured members' funds without raising equity capital, are able to offer their members more profitable insurance protection than commercial insurers in certain cases. First of all, this is due to the lack of a commercial component in the MIS insurance premium and preservation of the funds of the insurance fund in the joint ownership of insurants [1, 2, 3].
However, the establishment of new MIS in the absence of the seed equity capital and with the existing acute competition in the market for insurance services appears to be difficult: such a society should accumulate an insurance fund in the short term, with a sufficiently high level of financial stability and satisfying all supervisory requirements, while preserving the economic attractiveness of membership for insurants [4, 5].

This problem can be solved through optimal distribution of initial and current insurance premiums of the society members by size and time periods. In this case, the schemes of mixed financing for MIS allow to decrease the financial burden on insurers if relatively inexpensive additional funds are raised from external sources [6, 7]. At the same time, different schemes are described by different levels of efficiency, which updates the subject of the research aimed at identifying the most acceptable options for insurers.

This article reviews a problem of finding optimal strategies for forming the MIS fund with a homogeneous composition of members from the standpoint of the insurer, given the level of its financial stability, including the possibility of raising additional financing. Due to the high complexity of finding an analytical solution determined by the stochastic nature of accumulation and expenditure of the fund’s resources, the problem is solved numerically using the methods of simulation modeling [8].

2. METHODS

Let’s consider an association of \( N \) homogeneous economic agents, each of which has an individual risk of loss in year \( t \), determined by a random variable \( q^n_t \), where \( n = 1, N, t = 1, T \). To simplify further calculations, let’s consider these random variables to be independent and equally distributed in the association of agents and time.

Let’s assume that members of the association decide to establish a MIS for \( q^n_t \) risk insurance for \( T \) years at a point in time \( t = 0 \). Let’s introduce the following assumptions regarding the MIS operation.

1. The MIS fund is replenished annually by each member contributing a certain insurance premium \( n^t = r^t \) (equal for all members due to the homogeneity of risks). Aside from the current insurance premium, value \( r^1_N = r^1 \) also includes the component of the member’s entry fee.
2. The MIS is fully liable for losses \( q^n_t \).
3. Maintaining the MIS operation entails certain costs \( C^t \). Due to the constancy of the MIS insurance portfolio volume within the model under consideration, we will consider the level of expenses \( C^t = C \) unchanged in time.
4. The funds accumulated by the fund are paid back to the insurers in equal shares after the expiration of the period of \( T \) years.

Taking these assumptions into account, following the results of the \( t \)-th year of operation, the fund’s balance \( H^t \) at \( t \leq T \) is determined by the following expression:

\[
H^t = \sum_{t=1}^T n^t r^t - \sum_{t=1}^T Q^t - tC,
\]

(1)

Where value \( Q^t = \sum_{n=1}^N q^n_t \) represents the total amount of payments per year \( t \).

Sequence of values \( \{H^t\}_{t=1,T} \) is a random process of funds accumulation by the MIS fund, governed by parameters \( \{r^t\}_{t=1,T} \).

Let’s find the stability of the insurance institution as its ability to be liable for its debts. A natural measure of stability is probabilities with which it can or cannot do so.
Let’s introduce a random value $\tau''$ equal to the moment of the alleged MIS default, which can be defined as the smallest value of $t$ at which the amount of funds of the insurance fund turns out to be negative:

$$\tau'' = \inf\{t: H^t < 0\}. \quad (2)$$

Then the probability that the fund won’t default over the period $T$ can be defined as the probability that value $\tau''$ exceeds value $T$:

$$P = \Pr\{\tau'' > T\}. \quad (3)$$

Probability (3) will be used as a measure of the MIS fund stability.

Let’s assess the feasibility of participation in the MIS operation and the efficiency of alternative strategies for the MIS fund formation, described by various options of distributing the values of the initial and current contributions of the insurant, on the basis of a criterion reflecting its discounted costs for $T$ years. The discount rate $d$ is set equal to the profitability of the insurant’s own funds.

In the absence of insurance, the aggregate loss of the insurant for $T$ years, estimated taking discounting into account, is a random value described by the following expression:

$$s_{n\text{insurance}}^n = \sum_{t=1}^{T} q_t^\alpha (1+d)^{t-1}. \quad (4)$$

Participation in the MIS operation in the context of a random nature of payments is described by incidental costs, the magnitude of which largely depends on the results of the entire MIS operation, including the possibility of the latter’s default. Without introducing additional assumptions on the procedure for distributing MIS liabilities in this case, let’s assume that there is a certain acceptable level of risk for the MIS default $\alpha$ for the insurant, at which they neglect the possibility of such an event.

Provided that the fund does not default over a period of time $[0, T]$, the efficiency of participating in its operation can be estimated by the conditional mathematical expectation of the member’s costs found as difference in the contributions $r^t$ and the random share of the member in the balance of the insurance fund at time $T$, taking discounting into account:

$$E[s_n^{\text{mutual}}|\tau'' > T] = \sum_{t=1}^{T} \frac{r^t}{(1+d)^{t-1}} - \frac{E[H^T(\tau'' > T)]}{N(1+d)^T}. \quad (5)$$

Reduction in the degree of the denominator per unit in the first term means that the insurance premium is paid before the beginning of the next year.

The ratio of the expected costs of insurance (5) to the mathematical expectation of losses in the absence of insurance (4) is used as a criterion for the efficiency of participating in the MIS for the insurant:

$$\frac{E[s_n^{\text{mutual}}|\tau'' > T]}{E[s_{n\text{insurance}}^n]} = u = \frac{E[s_n^{\text{mutual}}]}{E[s_{n\text{insurance}}^n]} \cdot \frac{\tau'' > T}{\tau''}. \quad (6)$$

Indicator (6) expresses the relative level of overpayment for risk insurance and can be used to compare various options for insurance protection.

The task of optimal management of the process of the MIS fund accumulation (1) in the conditions of its isolated operation can be formulated as follows: a strategy for the fund formation from initial and current contributions $\{r^t\}_{t=1}^{T} = \bar{r}$ must be defined, in which the cost of insurance protection $u$ for members is the least, with a restriction on the level of the fund stability $P$. 

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It can be shown that due to the monotonicity of functions \( u(\vec{r}) \) and \( P(\vec{r}) \) for each of the arguments, the solution of problem (7) exists and is located on the surface \( O = P^{-1}(1 - \alpha) \) (on the boundary of the acceptable area).

## 3. RESULTS

Let’s assume that the MIS can freely exchange financial flows \( f^t \) with some third-party organization (Center), which has a rate of return on its own funds \( g \neq d \), throughout the entire period of its operation. Let positive values of \( f^t \) correspond to cash flows from the Center to the MIS, and the negative values correspond to the cash flows from the MIS to the Center. Then the equation of the MIS fund accumulation process (1) can be represented as follows:

\[
H^t = \sum_{t=1}^{T} (N r^t + f^t) - \sum_{t=1}^{T} Q^t - t C.
\]

(8)

The net discounted income of the Center from participation in the MIS financing, if the fund does not default, will amount to:

\[
G(\vec{f}) = \sum_{t=1}^{T} \frac{-f^t}{(1+g)^{(T-t)}}.
\]

(9)

Let’s assume that the Center takes a decision to finance the MIS, if the income of the Center is greater than a certain constant value \( G(\vec{f}) \geq G_0 \). If \( G_0 > 0 \), the Center can be interpreted as a commercial financial institution. The case \( G_0 < 0 \) is possible if an agent pursuing some noncommercial interests plays the role of the Center. In particular, a state that provides support to MIS can be such an organization.

Given the availability of an additional source of financing, the optimization problem (7) can be modified as follows:

\[
\begin{align*}
\{ & u(\vec{r}, \vec{f}) \rightarrow \min \\
& P(\vec{r}, \vec{f}) \geq 1 - \alpha \\
& G(\vec{f}) \geq G_0 \\
& r^t \geq 0 \\
& \text{where } \vec{f} = \{f^t\}_{t=1,T}.
\end{align*}
\]

(10)

The criterion for this problem takes into account that flows \( f^t \) are obliquely included in the expression for the cost of insurance \( u(\vec{r}, \vec{f}) \) through the component \( E[H^T|\{r^t \geq T\}] \).

Due to the fact that the analytical way of finding dependencies \( P(\vec{r}) \) and \( u(\vec{r}) \) is extremely difficult, it is proposed to estimate their parameters when solving problems (7) and (10) on the basis of simulation modeling methods.

The following simulation experiment will be held: for each of the \( N \) members of the society the flows of random losses \( q^t_n \) will be simulated, on the basis of which samples of realizations of the MIS fund accumulation process (1) will be generated for fixed external parameters \( N, q^t_0, C \) and fixed control parameters \( r^t \). Taking these samples into account, the probability that fund \( P \) will not default is defined as the ratio of the number of realizations of process \( \{H^t\} \) that do not contain values \( H^t < 0 \) to the total number of experiments held, and the conditional mathematical expectation \( E[H^T|\{r^t \geq T\}] \) is defined as the mean value of \( H^T \) taken for these realizations.
Further, using the described algorithm for finding the values of functions $P(\bar{r})$ and $\mu(\bar{r})$, as well as the method of gradient descent, a numerical solution of problem (7) is obtained for a fixed value of parameter $\alpha$.

The flows of losses of the MIS members will be formed as follows when holding simulation experiments: losses of the $n$-th agent in the $t$-th year $q_t^n$ take place with probability $p$, and their value is defined by some random value $\pi$. As the probability distribution of the latter, it is proposed to use the exponential distribution characteristic of many cases, without loss of generality. Then the characteristics of individual risk can be defined as follows:

$$q_t^n = \theta \pi, \theta = \begin{cases} 1, & p \\ 0, & 1 - p \end{cases}, p\pi(x) = \lambda e^{-\lambda x}, x \geq 0. \quad (11)$$

A scheme of the simulation experiment similar to the case of isolated MIS is used to solve problem (10).

In each experiment held, $10^5$ realizations of the model were considered with the distribution parameters $q_t^n$ equal $p = 0.1, \lambda = 1$. The expected payments for one occurred insurance event $E[q_t^n | \theta = 1] = \lambda^{-1} = 1$ were taken as a unit of measurement of the amount of money.

In all the experiments, the number of agents is chosen equal $N = 50$, the planning period is $T = 10$ years, the discount rate corresponding to the return on agents’ equity is assumed to be equal to $d = 0.2$, the costs of maintaining the MIS operation $C = 0$. The acceptable level of default probability in most experiments is set in the range between 5% and 25% (using higher levels of stability is associated with a significant increase in the size of a sample required for calculating the quantities under consideration).

4. DISCUSSION

Numerical solutions of optimization problem (7) at different levels of the acceptable probability of default $\alpha$ indicate that for the source data under consideration, the best strategy in terms of ensuring the fund stability for its members is to contribute significant initial assets (2-3 times greater than the expected annual payments) to the MIS at the very beginning of its operation, with a relatively uniform annual replenishment of the fund in the future. According to this strategy, the level of the insurance premium since the second year of the MIS operation is in the range from 1.2 to 1.4 of expected annual payments on average, depending on the chosen level of the fund's stability $\alpha$.

The dynamics of the premium level is largely determined by two opposite consequences of the fund replenishment. Early contribution to the fund is more efficient in terms of stability but increases the cost of insurance due to the discount effect. The solutions of problem (7) with an acceptable probability of the MIS default $\alpha = 15\%$ for various levels of discount $d$ indicate that the optimal premium size can both decrease for low values of $d$ (moment of investment is indifferent) and locally grow at high values (delay of investment is preferable).

The use of a more complex procedure for the fund filling allows the insurant to save up to 20% of the cost of insurance, while maintaining the same level of stability.

The relative cost of insurance grows along with an increase in the discount coefficient: at high levels of return on equity, it is more advantageous for insurants to take risk than to deposit income-bearing funds into the insurance fund. At low levels of return on equity, the relative cost of insurance $\mu$ may be less than one (due to the exclusion of case of the fund default from calculation). In this case, participation in the MIS can be regarded as a risky investment tool.

Given the level of the fund stability, the dependence of the cost of insurance on the value of fixed costs can be described by the following approximating expression with a high degree of accuracy:
\[ u(C) = u(0) + \frac{c}{\text{NE}[S_{\text{insurance}}]} \sum_{t=1}^{T} \frac{1}{(1+d)^{(t-1)}} \]  

(12)

As a result of solving problem (10), the dynamics of flows \( r^t \) and \( f^t \) with the value of the rate of return of the Center's equity \( g = 0.1 \) and \( g = 0.25 \) was obtained. The required present income of the Center from participation in the MIS financing \( G_0 \) is set at zero.

If \( g < d \), it is advantageous for the MIS members to make the initial replenishment of the fund, raising borrowed funds from the Center with their subsequent return by raising the level of premiums in the last years of the fund's operation. As such, the Center acts as a credit institution.

If \( g > d \), the opposite strategy turns out efficient: to send a part of the premium flow to the Center and reduce the level of insurance premium in the last years through subsequent refunds. This model of interaction can be interpreted as investment of insurance reserves.

The cost of participation in the MIS is reduced for different values of rates \( g \) and \( d \), with their almost any acceptable levels and the corresponding nature of interaction of the MIS with the Center.

In the example provided, when the Center is ready to incur a loss in the amount of one expected payment on the MIS portfolio, the optimal value of the cost of insurance is reduced more than 1.5 times. Such interaction can be interpreted as preferential lending, or subsidizing.

With the values of the MIS operation parameters used in calculations, the Center can extract the present income in the amount of up to 2.7 volumes of expected payments on the MIS portfolio, without changing the cost of insurance and the fund stability.

5. CONCLUSION

At the initial stage of their operation, the MIS are described by relatively low financial stability due to the insufficiency of their equity and the complexity of increasing their funds by their members. In such conditions, the increase in the MIS attractiveness as a type of insurance protection is associated with the rationale for optimal strategies for the insurance fund formation.

The task of finding the optimal management of the process of MIS insurance fund accumulation can be solved by analytical methods. However, in the context of a significant number of random variables taken into account, with the known laws of their distribution, it is expedient to use methods of simulation for solving such problems in order to simplify calculations.

From the insurants' standpoint, the least expensive strategy of the isolated MIS fund formation from its equity at a given level of stability is the introduction of a large entry fee several times greater than the actuarially fair level of the current contributions, followed by annual fund replenishments with small payments.

The introduction of fixed costs of the MIS to the model increases the cost of insurance in proportion to their size, without affecting the dynamics of optimal payments.

The availability of an external source of the MIS funding allows to significantly reduce the costs of its members, which appear inversely proportional to the absolute difference in the rates of return on their capitals when implementing the optimal procedure of interaction between the MIS and that source.

The results of the experiments held indicate that mutually beneficial cooperation between the Center and the MIS with a high level of the fund stability is possible in the context of mixed financing.
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REFERENCES


