International Journal of Civil Engineering and Technology (IJCIET)
Volume 8, Issue 8, August 2017, pp. 1123–1129, Article ID: IJCIET_08_08_120
Available online at http://http://www.iaeme.com/ijciet/issues.asp?JType=IJCIET&VType=8&IType=8
ISSN Print: 0976-6308 and ISSN Online: 0976-6316

A NEW APPROACH FOR SOLVING LINEAR FUZZY FRACTIONAL TRANSPORTATION PROBLEM

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ABSTRACT

This paper deals with a solution procedure for solving linear fuzzy fractional transportation problem (LFFTP) which is a special type of linear fuzzy fractional programming problem (LFFPP). Fuzzy version of improved Vogel's approximation method and Fuzzy version of Modi method are used to obtain the fuzzy optimal solution to the given LFFTP by without reformulating the original problem into an equivalent crisp problem. Also a numerical example is discussed for supporting the solution theory developed in this paper.

Key words: Trapezoidal fuzzy number; Fuzzy ranking; Fuzzy fractional programming problem; Fuzzy fractional transportation problem.

Cite this Article: S. Mohanaselvi, K. Ganesan, A New Approach for Solving Linear Fuzzy Fractional Transportation Problem. *International Journal of Civil Engineering and Technology*, 8(8), 2017, pp. 1123–1129
http://www.iaeme.com/IJCIET/issues.asp?JType=IJCIET&VType=8&IType=8

1. INTRODUCTION

In various applications such as production planning, financial planning etc., the decision maker may be interested in optimizing an objective function having ratio of linear function. These types of problems can be handled by using linear fractional programming problem (LFPP) techniques. In literature LFPP was first developed and studied extensively by Matros.B et al (1960) [8]. Later on several authors such as Bajalinov, E. B.(2003) , Charnes and Cooper (1962), Odior, A. O.(2012), Pandey P and Punnen A P.(2007)[1, 3,14,15] proposed different approaches for solving LFPP. Transportation technique in LFPP was first introduced by Swarup.K (1966) [17]. Also Gupta V et al (1993) [5] studied about paradox in linear fractional transportation problems (LFTP) with mixed constraints. Joshi D et al (2011) [6] investigated the transportation problem with fractional objective function when the demand and supply quantities are varying. Moanta, D.(2007) [10] proposed a simplex method technique for solving three dimensional transportation problem whose objective function is the ratio of two positive linear functions. Nuran Guzel (2012)[13] presented an Taylor series approximation and interval arithmetic based procedure for the solution of interval fractional transportation problem. Bheeman Radhakrishnan (2014) [2] proposed a compensatory approach to LFFTP by using Werner's 'fuzzy or' operator. Narayanamoorthy.S et al (2015) [12] presented a solution procedure to solve LFFTP by using dual simplex method. Kalyani. S et al (2016) [7] converted the given LFFTP into two fuzzy transportation problem and presented the solution. By solving pair of linear programs at a specific α-cut Shiang-Tai Liu (2016) [16] discussed a solution procedure for LFFTP. V.A.Jadhav et al (2016) [5] presented a compensatory approach for LFFTP by solving each of the fractional objective function independently.

In this paper a solution procedure for solving fuzzy fractional transportation problem is discussed without reformulating the original problem into classical type. In section 2 preliminaries necessary for supporting our work is presented. In section 3 mathematical formulation of LFFTP is defined and an algorithm for solving it is presented. In section 4 a numerical example is discussed for supporting the above said algorithm. In section 5 conclusion of this paper is presented.

2. PRELIMANARIES

In this section, we exhibit some elemental definitions, which are used all through this paper.

Definition 1

A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\tilde{A}: R \to [0,1]$ has the following characteristics:

- $\tilde{A}$ is convex
- $\tilde{A}$ is normal
- $\tilde{A}$ is piecewise continuous.

Definition 2

A fuzzy number $\tilde{A}$ on R is said to be a trapezoidal fuzzy number (TrFN) if its membership function $\tilde{A}: R \to [0,1]$ has the following characteristics:

$$\tilde{A}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \le x \le a_2 \\ 1, & \text{for } a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & \text{for } a_3 \le x \le a_4 \\ 0, & \text{elsewhere} \end{cases}$$

We denote the trapezoidal fuzzy number by $\tilde{A} = (a_1, a_2, a_3, a_4)$.

Definition 3

A fuzzy number can also be expressed as a pair $\tilde{A} = \left[A^-(r), A^+(r)\right]$ of functions $A^-(r)$ and $A^+(r)$ for $0 \le r \le 1$ which satisfies the following requirements:

- $A^-(r)$ is a bounded monotonic increasing left continuous function.

- $A^{+}(r)$ is a bounded monotonic decreasing left continuous function.
- $A^{-}(r) \le A^{+}(r)$, $0 \le r \le 1$.

Definition 4
The trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ can be hence represented by $\tilde{A} = \left[A^{-}(r), A^{+}(r)\right] = \left[(a_2 - a_1)r + a_1, a_4 - (a_4 - a_3)r\right]$.

Definition 5
The midpoint of a fuzzy number $\tilde{A} = \left[A^{-}(r), A^{+}(r)\right]$, where $0 \le r \le 1$ is defined by $m_A = \left(\frac{A^{-}(1) + A^{+}(1)}{2}\right)$.

2.1. Arithmetic operation on Fuzzy Numbers:
Ming Ma et al [16] has proposed a new fuzzy arithmetic operator based on parametric form of fuzzy numbers. The following arithmetic operation on fuzzy numbers based on the parametric triplet $\tilde{A} = \left(m_A, m_A - A^{-}(r), A^{+}(r) - m_A\right)$ is used in this paper.

For any two fuzzy numbers $\tilde{A} = \left[A^{-}(r), A^{+}(r)\right]$ and $\tilde{B} = \left[B^{-}(r), B^{+}(r)\right]$ the arithmetic operations $* = \{+, -, \times, \div\}$, are defined as:

$$\tilde{A} * \tilde{B} = \left(m_A * m_B, \max\left\{m_A - A^{-}(r), m_B - B^{-}(r)\right\}, \max\left\{A^{+}(r) - m_A, B^{+}(r) - m_B\right\}\right)$$

2.2. Ranking of Fuzzy Numbers
In decision making problems ranking of fuzzy numbers is an essential part to make a best decision. In this paper the magnitude of a fuzzy number is calculated as follows to rank the fuzzy numbers.

$$\mathrm{Mag}(\tilde{A}) = \int_0^1 \left(\frac{A^{-}(r) + A^{+}(r)}{2} + m_A\right) r \, dr$$

Definition 1
Two fuzzy numbers $\tilde{A} = \left[A^{-}(r), A^{+}(r)\right]$ and $\tilde{B} = \left[B^{-}(r), B^{+}(r)\right]$ are said to be equivalent if and only if $\mathrm{Mag}(\tilde{A}) = \mathrm{Mag}(\tilde{B})$. That is $\tilde{A} \approx \tilde{B}$ if and only if $\mathrm{Mag}(\tilde{A}) = \mathrm{Mag}(\tilde{B})$. And they are said to be equal that is $\tilde{A} = \tilde{B}$ if and only if $m_A = m_B$, $m_A - A^{-}(r) = m_B - B^{-}(r)$ and $A^{+}(r) - m_A = B^{+}(r) - m_B$.

3. MATHEMATICAL FORMULATION OF LFFTP
The linear fuzzy fractional transportation problem (LFFTP) is a part of logistics and supply chain management problems for improving the profit of the organization while considering the other factors which affects the profit. Let there be m sources from which goods have to be supplied to n destinations. Let $\tilde{C} = \left(\tilde{c}_{ij}\right)_{m \times n}$ be the fuzzy cost matrix where $\tilde{c}_{ij}$ is the cost spend in transporting the goods from a source i to destination j. Let $\tilde{P} = \left(\tilde{p}_{ij}\right)_{m \times n}$ be the fuzzy profit matrix where $\tilde{p}_{ij}$ is the fuzzy profit gained if a unit of good is transported from a source i to

destination j. Let $\tilde{x}_{ij}$ be the number of unknown quantities of goods to be transported from the source i to the destination j. Let $\tilde{c}_0$ and $\tilde{p}_0$ be the given fixed fuzzy cost and fuzzy profit. Then the mathematical formulation of LFFTP is given by:

$$\text{Min } \tilde{Q}(x) \approx \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\,\tilde{x}_{ij} + \tilde{c}_0}{\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{p}_{ij}\,\tilde{x}_{ij} + \tilde{p}_0} \approx \frac{\tilde{C}(x)+\tilde{c}_0}{\tilde{P}(x)+\tilde{p}_0}$$

$$\begin{aligned} &\sum_{j=1}^{n} \tilde{x}_{ij} \preceq \tilde{b}_i \\ &\sum_{i=1}^{m} \tilde{x}_{ij} \succeq \tilde{a}_j \\ &\tilde{a}_j \approx \tilde{b}_i \\ &\tilde{x}_{ij} \succeq 0, \ \forall i,j \end{aligned} \qquad (1)$$

Since we are considering the fraction of linear fuzzy function it may be possible that for some $\tilde{x}_{ij} \succeq 0$ the denominator may be equal to zero. To avoid that situation we assume that always $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{p}_{ij}\,\tilde{x}_{ij} + \tilde{p}_0 \neq \tilde{0}$

Definition 1

A set of non-negative allocations $\tilde{x}_{ij}$ which satisfies the row and the column restrictions (in the sense equivalent) is known as fuzzy feasible solution to (1).

Definition 2

A fuzzy feasible solution to (1) is said to be a fuzzy basic feasible solution if the number of positive allocations made are $(m+n-1)$. If the number of allocations in a fuzzy basic feasible solution are less than $(m+n-1)$, then it is called as a fuzzy degenerate basic feasible solution.

Definition 3

A fuzzy basic feasible solution to (1) is said to be fuzzy optimal solution if it minimizes the objective function.

4. PROPOSED METHOD TO FIND THE FUZZY OPTIMAL SOLUTION TO THE GIVEN LFFTP

The solution procedure to obtain the fuzzy optimal solution to the given LFFTP involves two steps. In the first step the initial fuzzy basic feasible solution is obtained by using fuzzy version of improved Vogel's approximation method and in the second step the initial basic feasible solution is improved by using fuzzy version of Modi method to obtain the fuzzy optimal solution. Mustafa Sivri et al (2011)[11] has proposed a improved transportation algorithm for solving LFTP. We have extended this algorithm for LFTP with fuzzy numbers.

4.1. Fuzzy version of improved vogel's approximation method

(i) Calculate the difference between two lowest fuzzy costs $\tilde{c}_{ij}$ in all rows and columns. Similarly calculate the difference between two lowest fuzzy profits $\tilde{p}_{ij}$ in all rows and columns

(ii) Calculate the sum of the differences of $\tilde{c}_{ij}$ and $\tilde{p}_{ij}$ for each row and column.

(iii) Identify the row or column that has the greatest sum of the differences compared with all other sum of the differences.

(iv) Suppose that i^{th} row has the greatest sum of difference. Determine the row for which $\frac{\tilde{c}_{ij}}{\tilde{p}_{ij}}$ is minimum and make maximum allotment in it .

(v) Repeat the process until all goods in the sources are transported.

4.2. Optimality condition to a fuzzy Transportation Problem

After determining the initial fuzzy basic feasible solution by the proposed algorithm, we have to test the current initial fuzzy basic feasible solution for optimality by using fuzzy version of modified distribution method. Let $\tilde{u}_1', \tilde{u}_2', \ldots\ldots, \tilde{u}_m'$ and $\tilde{u}_1'', \tilde{u}_2'', \ldots\ldots, \tilde{u}_m''$ be the multipliers for fuzzy cost and fuzzy profit to the m constraints and let $\tilde{v}_1', \tilde{v}_2', \ldots\ldots, \tilde{v}_n'$ and $\tilde{v}_1'', \tilde{v}_2'', \ldots\ldots, \tilde{v}_n''$ be the multipliers for fuzzy cost and fuzzy profit to the n constraints. We can calculate $\tilde{u}_i', \tilde{u}_i''$ and $\tilde{v}_i', \tilde{v}_i''$ for the allocated cells using the relation $\tilde{c}_{ij}' = \tilde{u}_i' + \tilde{v}_i'$ and $\tilde{p}_{ij}'' = \tilde{u}_i'' + \tilde{v}_i''$ by setting a multiplier to zero which is associated with the row or column of the transportation table that contains the maximum number of allocated cells. If $\Delta_{ij}' = \tilde{c}_{ij} - \tilde{u}_i' - \tilde{v}_i'$ and $\Delta_{ij}'' = \tilde{p}_{ij} - \tilde{u}_{ij}'' - \tilde{v}_{ij}''$, then criterion for optimality is given by $\Delta_{ij} \succeq \tilde{0}$ where $\Delta_{ij} = \Delta_{ij}'' - \tilde{Q}(x)\Delta_{ij}' \quad \forall i, j$ for the unallocated cells of fuzzy fractional transportation table.

5. NUMERICAL EXAMPLE

Consider a numerical example discussed by [12]

$$\text{Min} \frac{(0,2,4,6)x_{11} + (1,2,6,7)x_{12} + (1,4,5,6)x_{21} + (3,4,5,8)x_{22}}{(0,1,3,4)x_{11} + (2,3,5,6)x_{12} + (1,3,5,7)x_{21} + (2,6,7,9)x_{22}}$$

$$x_{11} + x_{12} \le 60$$
$$x_{21} + x_{22} \le 45$$
$$x_{11} + x_{21} \ge 50$$
$$x_{11} + x_{22} \ge 55$$
$$x_{ij} \ge 0$$

Representing the given problem in parametric triplet and applying the algorithm explained in the previous section we have

Table 1 Fuzzy fractional transportation problem

$(3,3\text{-}2r,3\text{-}2r)$ $(2,2\text{-}r,2\text{-}r)$	$(4,3\text{-}r,3\text{-}r)$ $(4,2\text{-}r,2\text{-}r)$	60
$(4.5,3.5\text{-}3r,1.5\text{-}r)$ $(4,3\text{-}2r,3\text{-}2r)$	$(4.5,1.5\text{-}r,3.5\text{-}3r)$ $(6.5,4.5\text{-}4r,2.5\text{-}2r)$	45
50	55	105

The difference between the lowest fuzzy cost cells for both $\tilde{c}_{ij}$ and $\tilde{p}_{ij}$ in 1st row is given by $(1,3\text{-}r,3\text{-}r)$ and $(2,2\text{-}r,2\text{-}r)$. Then their sum of differences is given by $(3,3\text{-}r,3\text{-}r)$. Similarly for the 2nd row the sum of the differences is given by $(2.5, 4.5\text{-}4r, 3.5\text{-}3r)$. For 1st column the sum of the differences is given by $(3.5, 3.5\text{-}3r, 3\text{-}2r)$ and 2nd column sum of the differences is given by $(3, 4.5\text{-}4r, 3.5\text{-}3r)$. The greatest sum of the differences compared with all other sum of the differences is available in the 1st column. Hence finding $\min[(1.5, 3\text{-}2r, 3\text{-}2r), (1.125, 3.5\text{-}3r, 3\text{-}2r)]$ we have the first allocation 45 in 2nd row 1st position. Similarly all other allocations are made.

Table 2 Fuzzy optimal solution to the given LFFTP

$(3, 3\text{-}2r, 3\text{-}2r)$ [5] $(2, 2\text{-}r, 2\text{-}r)$	$(4, 3\text{-}r, 3\text{-}r)$ [55] $(4, 2\text{-}r, 2\text{-}r)$	60
$(4.5, 3.5\text{-}3r, 1.5\text{-}r)$ [45] $(4, 3\text{-}2r, 3\text{-}2r)$	$(4.5, 1.5\text{-}r, 3.5\text{-}3r)$ $(6.5, 4.5\text{-}4r, 2.5\text{-}2r)$	45
50	55	105

Hence the initial fuzzy basic feasible solution (IBBFS) to the given LFFTP is obtained as $x_{11} = 5,\ x_{12} = 55$ and $x_{21} = 45$.

$$\tilde{Q}(x) = \frac{(437.5, 3.5 - 3r, 3 - 2r)}{(410, 3 - 2r, 3 - 2r)} = (1.067, 3.5 - 3r, 3 - 2r).$$

Now we will check whether the IFBFS obtained is optimal or not by using the improved fuzzy Modi method. By assuming $\tilde{u}_1' = \tilde{0}$ we calculate $\tilde{u}_2', \tilde{v}_1', \tilde{v}_2'$ for the allocated cells for the fuzzy cost matrix. Similarly by assuming $\tilde{u}_1'' = \tilde{0}$ we calculate $\tilde{u}_2'', \tilde{v}_1'', \tilde{v}_2''$ for the allocated cells for the fuzzy profit matrix. Now we calculate $\Delta_{22}' = \tilde{c}_{22} - \tilde{u}_2' - \tilde{v}_2' = (-1.5, 3.5 - 3\text{r}, 3.5 - 3\text{r})$ and $\Delta_{22}'' = \tilde{p}_{22} - \tilde{u}_{22}'' - \tilde{v}_{22}'' = (0.5, 3.5 - 3\text{r}, 3.5 - 3\text{r})$ for the unallocated cell. The criteria for optimality $\Delta_{ij}'' - \tilde{Q}(x)\Delta_{ij}' \succeq \tilde{0}$ are satisfied. Hence the obtained fuzzy initial fuzzy basic feasible solution is optimal.

$$\text{Min } \tilde{Q}(x) = (1.067, 3.5 - 3r, 3 - 2r)$$

6. CONCLUSIONS

In this paper a direct method for solving a linear fractional transportation problem with fuzzy coefficient is considered. Without reformulating the original problem into a classical problem and by using the mentioned fuzzy arithmetic and fuzzy ranking technique we have obtained the fuzzy optimal solution to the given LFFTP. A numerical example is illustrated for describing the solution procedure explained in this paper.

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