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# NEW SCHEME FOR ONE DIMENSIONAL WAVE EQUATION

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**ABSTRACT**

*In this research article, it is introduced that a scheme to obtain a numerical solution for one dimensional wave equation. Also, we compared this scheme with the existing scheme (explicit scheme). It is observed that this scheme gives the better results.*

**Keywords**: Crank Nicolson scheme, stability, wave equation

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## 1. INTRODUCTION

Many problems in science and engineering can be formulated in either initial value problems or boundary value problems [1, 2, 3]. Consider general second order linear partial differential equation is of the form.

$$A(x,y)u_{xx} + B(x,y)u_{xy} + C(x,y)u_{yy} + D(x,y)u_x + E(x,y)u_y + F(x,y)u + G(x,y) = 0 \quad (1.1)$$

Equation (1.1) is called hyperbolic if $B^2 - 4AC > 0$. An example of a partial differential equation is one dimensional wave equation for the amplitude function $u(x,t)$ as

$$Au_{xx} = u_{tt} \text{ For } 0 \le x \le l;\ 0 \le t \le T \quad (1.2)$$

Where $x$ is position and $t$ is time In order to solve this equation, the boundary conditions $u(0,t) = 0$; $u(l,t) = 0$, as well as the initial conditions $u(x,0) = f(x)$; $u_t(x,0) = 0$ should be provided.

## 2. THE NUMERICAL SCHEME FOR SOLVING 1-DIMENSIONAL WAVE EQUATION

### a) Existing scheme:

The solution for one dimensional wave equation (1.2) by replacing the corresponding central finite difference relations

$$u_{xx} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} \tag{2.1}$$

$$u_{tt} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} \tag{2.2}$$

Is identified by the relation (Explicit scheme)

$$u_{i,j+1} = 2\left(1-\lambda^2 a^2\right)u_{i,j} + \lambda^2 a^2\left(u_{i-1,j} + u_{i+1,j}\right) - u_{i,j-1} \tag{2.3}$$

Where, $\lambda = \frac{k}{h}$. To make the equation (2.3) simpler form, choose $\lambda$ such that $1-\lambda^2 a^2 = 0$, then equation (2.3) reduces to

$$u_{i,j+1} = \left(u_{i-1,j} + u_{i+1,j}\right) - u_{i,j-1} \tag{2.4}$$

Equation (2.4) is the simpler form of (2.3) under the condition $k = \frac{h}{a}$ and it enables us to find $u$ at the $(j+1)^{th}$ row only if we know the values of $u$ at $j^{th}$ and $(j-1)^{th}$ row. The following is the schematic representation of the scheme (2.4)

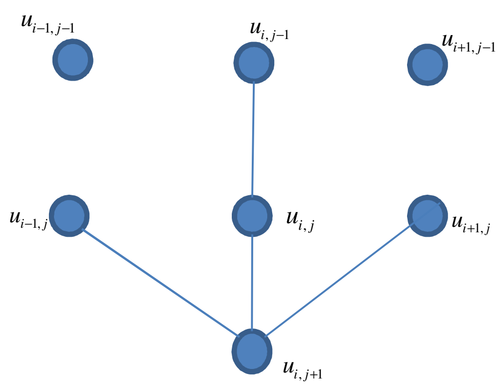


**Figure 2.1** Schematic representation of explicit scheme

## b) Proposed scheme:

Here we proposed a new scheme for solving one dimensional wave equation with respect to the corresponding initial and boundary conditions. Replacing $u_{xx}$ and $u_{tt}$ in (1.2) with their respective finite difference approximations; i.e.

$$u_{xx} = \frac{u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1} + u_{i+1.j} - 2u_{i,j} + u_{i-1,j}}{2h^2} \tag{2.5}$$

$$u_{tt} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} \tag{2.6}$$

We get,

$$u_{i,j+1} = \frac{a^2\lambda^2}{2}\left(u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1} + u_{i+1,j} + u_{i-1,j}\right) + u_{i,j}\left(-a^2\lambda^2 + 4\right) - 2u_{i,j-1} \tag{2.7}$$

Where, $\lambda = \frac{k}{h}$

Equation (2.7) is our proposed scheme and named as Murty's scheme

When, $\lambda = \frac{2}{a}$, we have the following

$$u_{i,j+1} = \frac{1}{5}\left[u_{i+1,j+1} + u_{i-1,j+1} + u_{i+1,j} + u_{i-1,j} - u_{i,j-1}\right] \qquad (2.8)$$

We identified that this scheme is valid for any positive value of $\lambda$

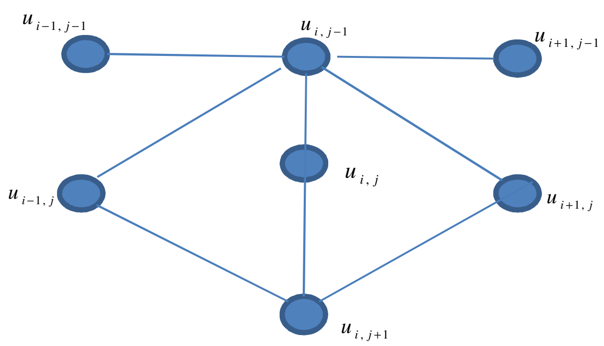


**Figure 2.2** Schematic representation of proposed explicit scheme

## 3. STABILITY

A numerical method is said to be stable if the cumulative effect of all the errors is bounded independent of the number of mesh points. We now examine the stability of the proposed scheme. In equation (2.7), if $4 - a^2\lambda^2 < 0$, i.e., $\lambda a > 2$ i.e., $\frac{ka}{h} > 2$, then the solution is unstable. If, $\frac{ka}{h} = 2$, then the solution is stable. That is, the solution is stable for $\lambda = \frac{2}{a}$

## 4. NUMERICAL RESULTS

In this section, we verified the results by the following:

Example: Consider the partial differential equation $4u_{xx} = u_{tt}$ with $u(0,t) = 0$, $u(4,t) = 0$, $u_t(x,0) = 0$, $u(x,0) = x(4-x)$ for four time steps.

Here we are providing the numerical solutions of example (1) with reference to the schemes 2(a), 2(b) and method of analytical solution as follows.

### Result by using analytical method:

| $t \backslash x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 3 | 4 | 3 | 0 |
| 1 | 0 | 2 | - | - | 0 |
| 2 | 0 | -3 | -4 | -3 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 3 | 4 | 3 | 0 |

**Table 1**

**Result by using existing explicit scheme:**

| $t\backslash x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 3 | 4 | 3 | 0 |
| 1 | 0 | 2 | 3 | 2 | 0 |
| 2 | 0 | 0.626 | 0.1304 | 0.026 | 0 |
| 3 | 0 | -0.50852 | -0.6730 | -0.50852 | 0 |
| 4 | 0 | -0.3270 | -0.3363 | -0.2070 | 0 |

**Table 2**

**Result by using existing proposed scheme:**

| $t\backslash x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 3 | 4 | 3 | 0 |
| 1 | 0 | 2 | - | - | 0 |
| 2 | 0 | -0.37804 | -0.49631 | -0.37763 | 0 |
| 3 | 0 | 0.00615 | 0.001086 | 0.000614 | 0 |
| 4 | 0 | 0.37804 | 0.496312 | 0.377633 | 0 |

**Table 3**

## 5. COMPARISON ANALYSIS:

Proposed method gives better results than existing explicit numerical method. The solutions which are obtained with proposed scheme are much closer to the solutions obtained analytically. The results are provided in tables (1)-(3).

## 6. CONCLUDING REMARKS:

In this article, it is proposed a new scheme for solving one dimensional wave equation subject to the conditions and compared the results with existing methods.

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