International Journal of Civil Engineering and Technology (IJCIET)
Volume 8, Issue 8, August 2017, pp. 926–936, Article ID: IJCIET_08_08_098
Available online at http://http://www.iaeme.com/ijciet/issues.asp?JType=IJCIET&VType=8&IType=8
ISSN Print: 0976-6308 and ISSN Online: 0976-6316
 Scopus Indexed

SPATIOTEMPORAL, STOCHASTIC DYNAMICS AND BIONOMIC ANALYSIS OF AN ECOLOGICAL SYSTEM WITH HARVESTING

A V S N Murty, M. N. Srinivas
Department of Mathematics, VIT University,
Vellore – 632014, Tamilnadu, India

D.R.V.S.R.K. Sastry
Department of Mathematics, Sastra University,
Thanjavur – 613401, Tamilnadu, India

J. Venkateswara Rao
Department of Mathematics, Mekelle University, Mekelle, Ethiopia

ABSTRACT

The present investigation is about the stability analysis of an ecological model. In this article, spatial heterogeneity model is proposed and discussed with and without spatiotemporal oscillations. Qualitative analysis and steadiness near inner steady state and spatiotemporal, noise changing aspects of the proposed model are also studied. It is also provided the computer simulations for verifying the results.

Key words: Prey, Predator, Harvesting, Local Stability Global Stability, Noise, Diffusion.

Cite this Article: A V S N Murty, M. N. Srinivas, D.R.V.S.R.K. Sastry and J. Venkateswara Rao, Spatiotemporal, Stochastic Dynamics and Bionomic Analysis of An Ecological System with Harvesting International Journal of Civil Engineering and Technology, 8(8), 2017, pp. 926–936.
http://www.iaeme.com/IJCIET/issues.asp?JType=IJCIET&VType=8&IType=8

1. INTRODUCTION

Now a day's population dynamics plays a major role in all aspects. Lotka and Volterra [11, 20] introduced research work on theoretical ecology. Ecological replicas have been in the attention of environmental discipline as predation of interrelating species, have emotional impact on population dynamics importantly. Review on steadiness of mechanism and the concept of spatial pattern construction through transmission driven instability of a system of intermingling populations in which a nonlinear Structure is asymptotically steady in the non-presence of transmission, but unsteady in the presence of diffusion show important role in bio mathematics and other areas of science [2], [3], [4], [13], [14], [17]. Spatial outlines alter the chronological dynamics and stability possessions of population concentrations at a variety of spatial scales, their consequences must be assimilated in chronological ecological replicas that do not signify space explicitly. And the spatial section of ecological interactions has been recognized as an significant feature in how environmental societies are shaped [14], [6]. In the spatial heterogeneity system the response item and the diffusion element comes by reason of quest-elusion phenomenon predators hunting prey and prey absconding predators. In such a system, there is an affinity that the effects of convective and dispersive interactions on the stability of two species are studied [19], [7].The consequence of transmission on the spatiotemporal prey-predator model has been explored by many scientists to their insightful work [10], [18]. Recently, the consequence of self as well as cross-diffusion in diffusion systems has established much attention by both environmentalists and theoreticians also. Balram Dubey et.al.,[5], motivated us to do this diffusion analysis of the proposed model.

2. MODEL EQUATIONS

In this section, the spatial heterogeneity system with harvesting and diffusion is proposed and investigated. The proposed mathematical model is given by

$$x_t = a_1 x - a_{11} x^2 - a_{12} xy - q_1 E_1 x + D_x x_{uu} \tag{2.1}$$

$$y_t = a_2 y - a_{22} y^2 + a_{21} xy - dy - q_2 E_2 y + D_y y_{uu} \tag{2.2}$$

where $x(t)$ is thickness of prey species, $y(t)$ is thickness of marauder species, a_1 is inherent growing rate of victim species, a_{11} is self- contact constant for victim class, a_{12} is amount of predation due to victim species, a_2 is inherent growing rate of killer species, a_{22} is self-interface constant for killer species, a_{21} is rate of predation due to killer, q_1 is catch capability coefficient of victim species, q_2 is catch capability coefficient of killer species, E_1 is exertion to yield the victim populaces, E_2 is exertion to yield the prey populace and d is demise rate of killer species, D_x, D_y represents the diffusive coefficients for victim, killer populaces.

3. ANALYSIS OF MODEL SYSTEM (2.1)-(2.2)

Without considering diffusion on the growth of the species, the model system (2.1) - (2.2) reduces to

$$x'(t) = x(a_1 - a_{11}x - a_{12}y - q_1 E_1) \tag{3.1}$$

$$y'(t) = y(a_2 - a_{22}y + a_{21}x - d - q_2 E_2) \tag{3.2}$$

For stability analysis, we assume that $a_1 - q_1 E_1 > 0; a_2 - d - q_2 E_2 > 0$ (3.3)

Study of Equilibrium points

From (3.1)-(3.2) it is clearly known to us, there exists 4 states as $E_0(0,0)$, $E_1(\overline{x},0)$, $E_2(0, y^{\phi})$ and $E_3(x^*, y^*)$. It is obvious that, the steady state $E_0(0,0)$ always exist. In the nonappearance of killer species, the stable point is $E_1(\overline{x},0) = E_1((a_1 - q_1 E_1)/a_{11}, 0)$. In the nonappearance of victim species, the stable point $E_2(0, y^{\phi}) = E_2[0, (a_2 - d - q_2 E_2)/a_{22}]$. In the occurrence of both victim and killer species the possible steady state is given by $E_3\left(x^*, y^*\right)$ where $x^* = \left(a_{22}\left(a_1 - q_1E_1\right) - a_{12}\left(a_2 - d - q_2E_2\right)\right) / \left(a_{11}a_{22} + a_{12}a_{21}\right)$

and $y^* = \left(a_{22}\left(a_1 - q_1E_1\right) + a_{11}\left(a_2 - d - q_2E_2\right)\right) / \left(a_{22}a_{12} + \alpha_{11}a_{22}\right)$. (3.4)

Here x^*, y^* are positive as per assumption. Here we are providing the page plots of the given ecological system (2.1)-(2.2) and graph phase planes Figures (3)(a) and (3)(b) of the system without variable length arrow and with variable length arrows.

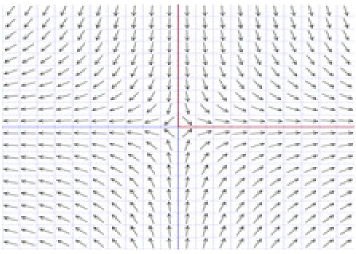


Figure 3(a)

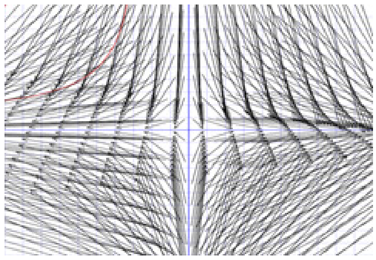


Figure 3(b)

Stability analysis

It is nothing but the steadiness of the given ecological system. Since the equilibrium points E_0, E_1, E_2 are unstable, the present study is restricted to the inner steady state. Now, we analyse the steadiness, in specific local one. To chequered the local steadiness, it is mandatory to custom the typical equation of variational matrix for the system (3.1)-(3.2) around $E_3\left(x^*, y^*\right)$ is $\lambda^2 + A_1\lambda + B_1 = 0$,

where $A_1 = a_{11}x^* + a_{22}y^*; B_1 = \left(a_{11}a_{22} + a_{12}a_{21}\right)x^*y^*$ (3.4)

Now the stability of the given system (3.1)-(3.2) purely depends on the signs of Eigen values (roots of equation (3.4)), $\lambda_1 + \lambda_2 = -\left(\alpha_{11}x^* + \alpha_{22}y^*\right) < 0$ and $\lambda_1\lambda_2 = \left(\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21}\right)x^*y^* > 0$, consequently $E_3\left(x^*, y^*\right)$ is indigenously asymptotically firm. Now let us define (or) construct the Lyapunov function to chequer the universal firmness of the structure (3.1)-(3.2) near $E_3\left(x^*, y^*\right)$.

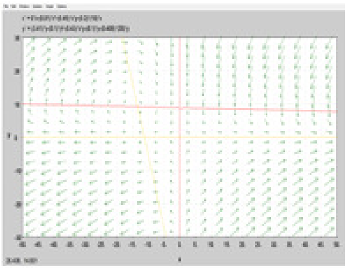
Let $V(x, y) = [x - x^* - x^* ln(x / x^*)] + l_1[y - y^* - y^* ln(y / y^*)]$, where l_1 a positive constant.

Differentiating V with respect to time t we get,

$$V'(t) = \left((x - x^*) / x\right)x'(t) + l_1\left((y - y^*) / y\right)y'(t)$$

$$V'(t) = -a_{11}\left(x - x^*\right)^2 - a_{12}\left(x - x^*\right)\left(y - y^*\right) + l_1\left(-a_{22}\right)\left(y - y^*\right)^2 + l_1a_{21}\left(x - x^*\right)\left(y - y^*\right)$$

by choosing $l_1 = a_{12} / a_{21}$, $V'(t) = -a_{11}\left(x - x^*\right)^2 - \left(a_{12}a_{22} / a_{21}\right)\left(y - y^*\right)^2 < 0$. Therefore $E_3(x^*, y^*)$ is globally asymptotically stable.



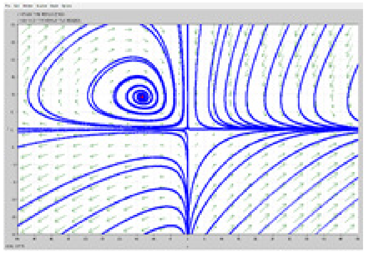
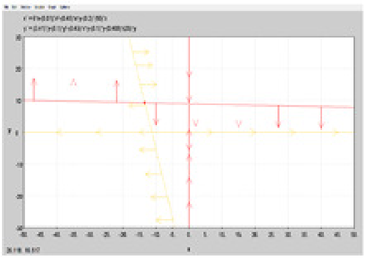


Figure 3(c) **Figure 3(d)**

Figures 3(c) and 3(d) represents nullclines and equilibrium point, plotting of stable and unstable orbits



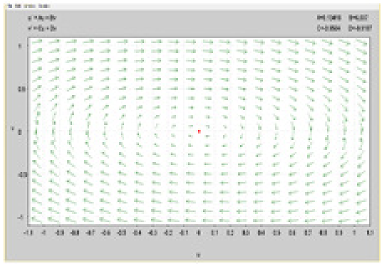


Figure 3(e) **Figure 3(f)**

Figures 3(c) and 3(d) represents nullclines with directions and spiral sink (-13.416, 9.187)

Bio economic equilibrium

Let us assume that c_1 and c_2 are fishing costs per unit effort for prey and predator species respectively. Let p_1, p_2 be the prices per unit biomass of prey, predator respectively. The Economic net revenue (or) Economic rent is given by $N = (p_1 q_1 x - c_1) E_1 + (p_2 q_2 y - c_2) E_2$ which is in the notation of $N = N_1 + N_2$ where $N_1 = p_1 q_1 x - c_1$ and $N_2 = p_2 q_2 y - c_2$.These two are represent the net revenues for prey & Predator respectively. The bio economic equilibrium $(x_\infty, y_\infty, (E_1)_\infty, (E_2)_\infty)$, is given by the following equations

$$x(a_1 - a_{11}x - a_{12}y - q_1 E_1) = 0 \quad (3.5)$$

$$y(a_2 - a_{22}y + a_{21}x - d - q_2 E_2) = 0 \quad (3.6)$$

$$N = (p_1 q_1 x - c_1) E_1 + (p_2 q_2 y - c_2) E_2 = 0 \quad (3.7)$$

In order to determine the bio economic equilibrium we now consider the following cases: Case (i) when the price is grander than the income for killer $(c_2 > p_2 q_2 y)$, then the killer harvesting will be clogged $(E_2 = 0)$, simply the victim harvesting remains operative $(c_1 < p_1 q_1 x)$, then $x_\infty = c_1 / (p_1 q_1)$; $y_\infty = c_2 / (p_2 q_2)$. Case (ii) when the price is grander than the income for victim $(c_1 > p_1 q_1 x)$, then the victim harvesting will be clogged $(E_1 = 0)$, simply the killer harvesting remains operative $(c_2 < p_2 q_2 y)$, then, $y_\infty = c_2 / (p_2 q_2)$; $x_\infty = (a_1 - q_1 E_1) / \alpha_{11}$. Case: (iii) If the cost is greater than the economic outcome of the both N_1 and N_2 $(c_1 > p_1 q_1 x; c_2 > P_2 q_2 y)$, then entire system will be collapsed $(E_1 = E_2 = 0)$. Case: (iv) If the economic outcome for both species N_1 and N_2 being positive $(c < p_1 q_1 x\,; c_2 < p_2 q_2 y)$

and the whole fishing will be in operation. In this case $(x)_\infty = c_1/(p_1q_1)$; $(y)_\infty = c_2/(p_2q_2)$; $(E_1)_\infty = \left[a_1 - a_{11}x^* - a_{12}y^*\right]/q_1$; $(E_2)_\infty = \left[a_2 - a_{22}y^* + a_{21}x^* - d\right]/q_2$. For $(E_1)_\infty, (E_2)_\infty$ are to be optimistic if $a_1 > a_{11}x^* + a_{12}y^*$ & $a_2 + a_{21}x^* > a_{22}y^* + d$ (3.8).Accordingly, the non-zero bionomic steady state point ($(x)_\infty, (y)_\infty, (E_1)_\infty, (E_2)_\infty$) occurs if the ailments (3.8) occurs.

4. DYNAMIC FORCES OF THE EXEMPLARY STRUCTURE (2.1)-(2.2)

The current research article deals with a class of extended prey predator systems in environmental science, modelled by diffusion equations. Although the dispersal system is a relatively simple model for the raid of prey species by predators in a spatial domain, the solutions exhibit an extensive spectrum of ecologically pertinent behaviour. Spatiotemporal dynamics includes chaos, target patterns [8], [9], [12]. The study of such spatiotemporal dynamics is an intensive area of research and there are still many unanswered questions concerning these solution types [12, 16].The spread of the population is observed by the pattern. These are two kinds of spread (i) The propagation of continuous travelling population fronts of high species density. (ii) The formation & movement of paths of high density separated by areas with density close to zero. The actual dynamics of the species spread is a result of the inter play between diffusion and deterministic factors. In this segment, we deliberated the exceptional influences of transmission of the ideal structure (2.1)-(2.2). $x=x(u,t), y=y(u,t)$, where u is a space variable and $x(u,0)>0;\ y(u,0)>0;$ for $u \in [0, \square]$. The trivial fluctuation edges conditions are specified by $\left[x_u\right]_{u=0,R} = 0; \left[y_u\right]_{u=0,R} = 0$. Now, let us consider the ideal (2.1)-(2.2) underneath trivial fluctuations edge ailments. To analyse the role of transmission on this ideal, we deliberate the linear ideal of the structure (2.1)-(2.3) about the interior steady state $E_3(x^*, y^*)$ as given by

$$X'(t) = -a_{11}x^*X - a_{12}x^*Y + D_xX_{uu} \tag{4.1}$$

$$Y'(t) = a_{21}y^*X - a_{22}y^*Y + D_yY_{uu} \tag{4.2}$$

by putting $x = x^* + X$; $y = y^* + Y$;. Assume the solutions of equations (4.1)-(4.2) in theform

$$X = \alpha_1 e^{\lambda t}\cos pu; Y = \alpha_2 e^{\lambda t}\cos pu \tag{4.3}$$

where p is the wave numeral of perturbation, λ is the frequency numeral & $\alpha_i, i = 1,2,3$ are the amplitudes. The characteristic equation of (4.1)-(4.2) is (using (4.3))

$$\mu^2 + A\mu + B = 0 \tag{4.4}$$

where $A = a_{11}x^* + a_{22}y^* + p^2(D_x + D_y)$;

$B = (a_{11}a_{22} + a_{12}a_{21})x^*y^* + (a_{11}D_yx^* + a_{22}D_xy^*)p^2 + p^4D_xD_y$

Theorem (1): The point $E_3(x^*, y^*)$ is locally asymptotically stable in the attendance of transmission if $a_{11}x^* + a_{22}y^* + p^2(D_x + D_y) > 0$ and $a_{11}a_{22}x^*y^* + (a_{11}D_yx^* + a_{22}D_xy^*)p^2 + p^4D_xD_y > 0$.

The above statement follows immediately by R-H criteria.

Theorem (2): (i) The system in the absence of spatiotemporal attributes at the inner steady state $E_3(x^*, y^*)$ attains steadiness, then the corresponding uniform steady state of the model

(2.1)-(2.2) in the presence of spatiotemporal attributes also attains steadiness. (ii) If the inner steady state $E_3(x^*, y^*)$ of the non-spatial heterogeneity system is unstable, then the respective steady state of the spatiotemporal model (2.1)-(2.2) under initial and boundary settings and attain steadiness by increasing or decreasing the spatiotemporal attributes suitably.

Proof: Let us define the function $V_l(t) = \int_0^R V(x, y)du$, where $V(x, y)$ is defined in Stability analysis section. Differentiating V_l w.r.t to t along the solutions of the diffusive model (2.1)-(2.2), we get,

$$V_l'(t) = \int_0^R (V_x x_t + V_y y_t)du = I_R + I_D \tag{4.7}$$

where $I_R = \int_0^R V'(t)dx$; $I_D = \int_0^R (D_x V_x x_{uu} + D_y V_y y_{uu})du$ (4.8)

Using the analysis in [14], we get $I_D = -D_x \int_0^R V_{xx}(x_u)^2 du - D_y \int_0^R V_{yy}(y_u)^2 du$

$$= -D_x \int_0^R (x^* / x^2)(x_u)^2 du - D_y \int_0^R (y^* / y^2)(y_u)^2 du \tag{4.9}$$

From (4.7), (4.8) and (4.9), it can clearly be observed that if $I_R < 0$ then $V_l'(t)$ is negative. If $I_R > 0$, then it is clearly showing if there is an increase in the spatiotemporal attributes D_x and D_y adequately huge numeral, $V_l'(t)$ as negative. Henceforth the succeeding portion of the theorem grasps.

5. STOCHASTIC ANALYSIS

Ecological systems are characteristically forced by a number of drivers such as climate and natural disturbances that are not constant in time but fluctuate. With the exception of processes dominated by deterministic oscillations, a significant part of environmental variability is random because of the uncertainty intrinsic in weather patterns, climate fluctuations and episodic disturbances like earth quakes, landslides, fires, insect outbreaks etc,.The recurrence of random drivers in bio-geophysical processes motivates the study of how a stochastic environment may affect and determine the dynamics of natural systems. We now extend the deterministic model (3.1)-(3.2) to analyse the role of random environmental fluctuations on stability. The random fluctuations make the parameters of the model to oscillate about their average values. We consider such randomness to the model (3.1)-(3.2) by incorporating additive white noises. The white noise perturbation included will change any parameter ν of the model as $\nu+\eta\psi(t)$, where η is the amplitude of the noise and $\psi(t)$ is a Gaussian white noise process at time t, but the deterministic and stochastic models have same equilibria which will also now fluctuate about their mean states. By considering the randomly fluctuating driving forces in the form of additive noise to the model (3.1)-(3.2), we get the following stochastic model

$$x'(t) = x(a_1 - a_{11}x - a_{12}y - q_1E_1) + \alpha_1\xi_1(t) \tag{5.1}$$

$$y'(t) = y(a_2 - a_{22}y + a_{21}x - d - q_2E_2) + \alpha_2\xi_2(t) \tag{5.2}$$

where α_1, α_2 are real constants and $\xi(t) = [\xi_1(t), \xi_2(t)]$ is a two dimensional Gaussian white noise process agreeable $E[\xi_i(t)] = 0;\ i = 1,2$; $E[\xi_i(t)\xi_j(t')] = \delta_{ij}\delta(t-t');\ i, j = 1,2$, where δ_{ij} is the Kronecker symbol; δ is the delta-Dirac function. In this analysis, we focus on the dynamics of the model (5.1)-(5.2) at the interior equilibrium point $E_3(x^*, y^*)$ only according to the method introduced by Nisbet and Gurney [15] and Carletti [1]. Let $x(t) = u_1(t) + S^*;\ y(t) = u_2(t) + P^*$ and by considering only the consequence of linear stochastic perturbations. Hence the model (5.1)-(5.2) reduces to the following linear system $u_1'(t) = -a_{11}u_1(t)S^* - a_{12}u_2(t)S^* + \alpha_1\xi_1(t)$ (5.3)

$$u_2'(t) = a_{21}u_1(t)p^* - a_{22}u_2(t)P^* + \alpha_2\xi_2(t) \quad (5.4)$$

Taking the Fourier transform of (5.3) and (5.4) we get,

$$\alpha_1\tilde{\xi}_1(\omega) = (i\omega + a_{11}S^*)\tilde{u}_1(\omega) - a_{12}S^*\tilde{u}_2(\omega) \quad (5.5)$$

$$\alpha_2\tilde{\xi}_2(\omega) = -a_{21}p^*\tilde{u}_1(\omega) + (i\omega + a_{22}P^*)\tilde{u}_2(\omega) \quad (5.6)$$

The matrix form of (5.5) and (5.6) is $M(\omega)\tilde{u}(\omega) = \tilde{\xi}(\omega)$ (5.7)

where $M(\omega) = \begin{pmatrix} A(\omega) & B(\omega) \\ C(\omega) & D(\omega) \end{pmatrix}$; $\tilde{u}(\omega) = \begin{bmatrix} \tilde{u}_1(\omega) \\ \tilde{u}_2(\omega) \end{bmatrix}$; $\tilde{\xi}(\omega) = \begin{bmatrix} \alpha_1\tilde{\xi}_1(\omega) \\ \alpha_2\tilde{\xi}_2(\omega) \end{bmatrix}$;

$$A(\omega) = i\omega + a_{11}S^*; B(\omega) = -a_{12}S^*; C(\omega) = -a_{21}P^*; D(\omega) = i\omega + a_{22}P^* \quad (5.8)$$

Hence the solution of (5.7) is $\tilde{u}(\omega) = K(\omega)\tilde{\xi}(\omega)$, where $K(\omega) = [M(\omega)]^{-1}$ (5.9)

The solution components of (5.9) are given by $\tilde{u}_i(\omega) = \sum_{j=1}^{2} K_{ij}(\omega)\tilde{\xi}_j(\omega);\ i = 1,2$ (5.10)

The spectrum of u_i, $i = 1,2$ are given by $S_{u_i}(\omega) = \sum_{j=1}^{2} \alpha_j |K_{ij}(\omega)|^2;\ i = 1,2$. Hence the intensities of fluctuations in the variable u_i, $i = 1,2$ are given by

$$\sigma_{u_i}^2 = \frac{1}{2\pi}\sum_{j=1}^{2}\int_{-\infty}^{\infty} \alpha_j |K_{ij}(\omega)|^2 d\omega,\ i = 1,2$$

. From (5.10), we obtain

$$\sigma_{u_1}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty} \alpha_1 \left|\frac{D(\omega)}{|M(\omega)|}\right|^2 d\omega + \int_{-\infty}^{\infty} \alpha_2 \left|\frac{B(\omega)}{|M(\omega)|}\right|^2 d\omega\right\}; \quad \sigma_{u_2}^2 = \frac{1}{2\pi}\left\{\int_{-\infty}^{\infty} \alpha_1 \left|\frac{A(\omega)}{|M(\omega)|}\right|^2 d\omega + \int_{-\infty}^{\infty} \alpha_2 \left|\frac{C(\omega)}{|M(\omega)|}\right|^2 d\omega\right\}$$

where $|M(\omega)| = R(\omega) + iI(\omega)$; $R(\omega) = -\omega^2 + (a_{11}a_{22} - a_{12}a_{21})S^*P^*$; $I(\omega) = \omega(a_{11}S^* + a_{22}P^*)$

If we consider the noise effect on any one of the species, which is with either $\alpha_1 = 0$ or $\alpha_2 = 0$ then we have, if $\alpha_1 = 0$

$$\sigma_{u_1}{}^2 = \frac{\alpha_2(-a_{12}S^*)^2}{2\pi}\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}d\omega \quad ; \qquad \sigma_{u_2}{}^2 = \frac{\alpha_2(-a_{21}P^*)^2}{2\pi}\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}d\omega \quad ;$$

and if $\alpha_2 = 0$, then

$$\sigma_{u_1}{}^2 = \frac{\alpha_1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\omega^2+(a_{22}P^*)^2\right]d\omega \; ; \quad \sigma_{u_2}{}^2 = \frac{\alpha_1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{R^2(\omega)+I^2(\omega)}\left[\omega^2+(a_{11}S^*)^2\right]d\omega$$

The population variances point out the stability of population for smaller values of mean square fluctuations, while the larger values of population variances indicate the instability of the populations.

6. NUMERICAL SIMULATIONS:

In this division, we established the analytical findings through numerical simulations using MATLAB.

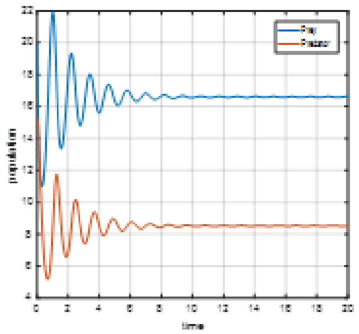


Figure 1

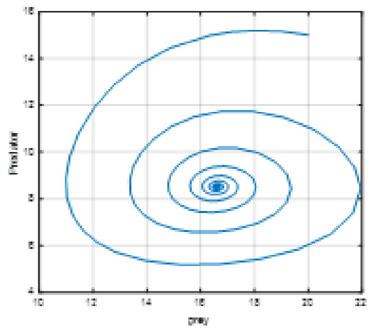


Figure 2

Figure1 signifies the variation of population against time when $a_{11} = 0.01;\ a_{12} = 0.45;\ q_1 = 0.2;\ a_2 = 3.41;$ $a_{22} = 0.1;\ a_{21} = 0.43;$ $d = 0.1;$ $q_2 = 0.408;\ E_1 = 10;\ a_1 = 6;\ E_2 = 20;$ & Figure 2 denotes the deviation of populace amongst victim and killer for the considerations of Figure (1)

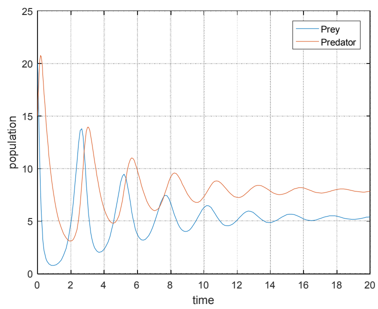


Figure 3

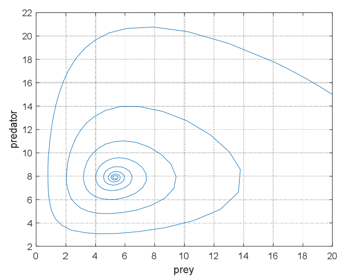


Figure 4

Figure 3 denotes the variation of populace against time when $a_1 = 6;\ a_{11} = 0.01;\ a_{12} = 0.5;\ q_1 = 0.2;$ $a_2 = 5;\ a_{22} = 0.05;\ E_1 = 10;\ E_2 = 15;\ a_{21} = 0.3;$ $d = 0.2;$ $q_2 = 0.4$; & Figure 4 denotes the deviation of population among victim and killer for the attributes of Figure 3.

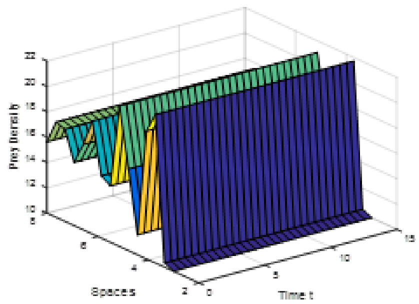


Figure 5

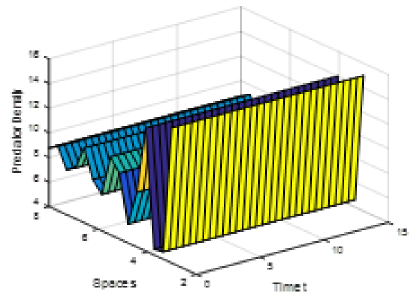


Figure 6

Figure 5 and Figure 6 denotes the steady fluctuations of the victim and killer populaces against space and time with $a_1 = 6; a_{11} = 0.01; a_{12} = 0.45; q_1 = 0.2; a_2 = 3.41;\ a_{22} = 0.1; a_{21} = 0.43; d = 0.1;$ $q_2 = 0.408;\ E_1 = 10;\ E_2 = 20;\ D_x = 10; D_y = 20$

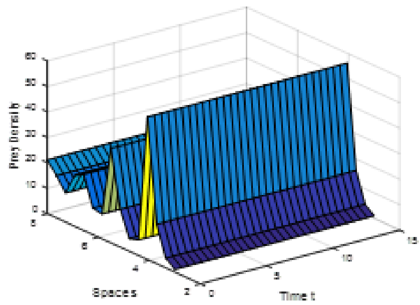


Figure 7

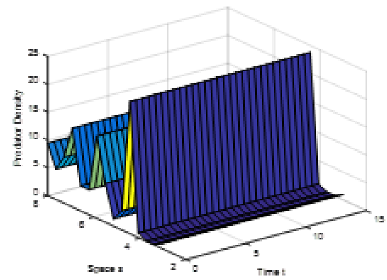


Figure 8

Figure 7 and Figure 8 denotes the steady fluctuations of the victim and killer populaces against space and time with $a_1 = 6;\ a_{11} = 0.01;\ a_{12} = 0.5;\ d = 0.2;\ q_2 = 0.4\ ;\ E_1 = 10;\ q_1 = 0.2;\ a_2 = 5;$ $a_{22} = 0.05;\ a_{21} = 0.3;\ E_2 = 15;\ D_x = 0.000001;\ D_y = 0.000002$

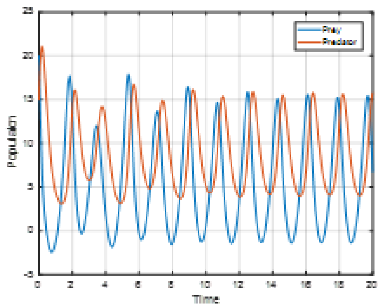


Figure 9

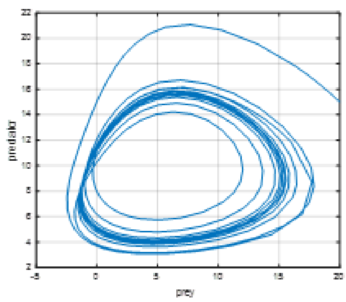


Figure 10

Figure 9 and Figure 10 shows that variation of species against time and phase portrait diagram of prey and predator respectively under random noise with $a_1 = 6; a_{11} = 0.01; a_{12} = 0.5;$ $d = 0.2;\ q_2 = 0.4\ ; E_1 = 10;\ q_1 = 0.2; a_2 = 5;\ a_{22} = 0.05;\ a_{21} = 0.3;\ E_2 = 15;\ \alpha_1 = 15; \alpha_2 = 3$

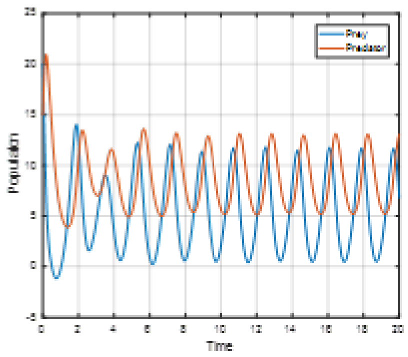


Figure 11

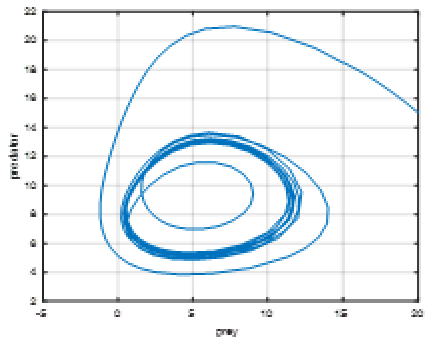


Figure 12

Figure 11 and Figure 12 shows that variation of species against time and phase portrait diagram of prey and predator respectively under random noise with $a_1 = 6; a_{11} = 0.01; a_{12} = 0.5;$ $d = 0.2;\ q_2 = 0.4\ ; E_1 = 10;\ q_1 = 0.2; a_2 = 5;\ a_{22} = 0.05;\ a_{21} = 0.3;\ \ E_2 = 15;\ \alpha_1 = 10; \alpha_2 = 1$

6. CLOSING OBSERVATIONS

In this, it is premeditated about a victim–killer ideal with harvesting and diffusion for both prey and predator which plays a major role in tuning the changing aspects of the model. We obtained all possible equilibrium points and analysed for stability using various mathematical techniques. Bio-economic conditions have been derived. It is shown that the dynamics of deterministic system in the Figures 1–4 and also analysed instability condition for diffusive structure of the ideal structure (2.1)-(2.2). Local and global stabilities are analysed using Routh-Hurwitz criteria and Lyapunov function respectively. It is also verified the stable oscillations of the prey and predator populations against time and space in figures 5-8. Also, the variation of species against time and phase portrait diagrams with noise effect in figures 9-12.

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