DIMENSIONAL ANALYSIS DESIGN MODEL OF BIOCHEMICAL OXYGEN DEMAND IN INTEGRATED SOLAR AND HYDRAULIC JUMP ENHANCED WASTE STABILIZATION POND

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ABSTRACT

A dimensional analysis design model for the prediction of biochemical oxygen demand (BOD) in the integrated solar and hydraulic jump enhanced waste stabilization pond (ISHJEWSP) was derived using the Buckingham’s π-theorem approach. The concentration of effluent BOD was derived as a function of the influent concentration of BOD, intensity of solar radiation, influent algae concentration, inlet velocity, density of wastewater, characteristic length of the pond, detention time, dispersion coefficient and dissolved oxygen. The model was calibrated with the first 54 data set and due to paucity of empirical literatures, was verified with the remaining 54 data set obtained from the experimental ISHJEWSP. The verification of the model gave a good coefficient of correlation of \( R = 0.864 \) between the measured and calculated concentration of effluent BOD at a significance level of 0.05. The regressed slope, intercept and standard error obtained were 0.938, 5.564 and 8.162 respectively. The coefficient of determination (R squared) showed that 74.68% of the variation of the experimental results is explained by the developed model. Therefore, the prediction of BOD concentration in the ISHJEWSP would be possible from the developed model.

Key words: Dimensional Analysis; Model; BOD; Regression; ISHJEWSP
1. INTRODUCTION

Mathematical models are utilized to approximate various highly complex engineering, physical, environmental, social and economic phenomena [1]. Dimensional analysis offers a method for reducing complex physical problems to the simplest (that is, most economical) form prior to obtaining a quantitative answer [2]. In the past, dimensional analysis has been applied in the fields of aerodynamics, hydraulics, ship design, propulsion, heat and mass transfer, combustion, mechanics of elastic and plastic structures, fluid-structure interactions, electromagnetic theory, radiation, astrophysics, underwater and underground explosions, nuclear blasts, impact dynamics, and chemical reactions and processing [3–6], and also biology [7] and even economics [8]. There are different methods of dimensional analysis, one of which is the Buckingham’s π-theorem. The principles of Buckingham’s π-theorem were essentially in place in the early 20th century [9]. Most applications of dimensional analysis are not in question, no doubt because they are well supported by experimental facts [2]. Given that the usefulness of dimensional analysis cannot be overemphasized, it was therefore applied to the development of a design model for the prediction of biochemical oxygen demand in integrated solar and hydraulic jump enhanced waste stabilization pond.

The integrated solar and hydraulic jump enhanced waste stabilization pond (ISHJEWSP) is introduced as a new technology that incorporates solar reflector and the introduction of hydraulic jump through change in pond bed slope of the conventional waste stabilization pond. The essence is for the purpose of increasing the treatment efficiency of the conventional WSP and consequently, the reduction in land area requirement [10]. WSPs are limited in application by their large area requirement [11]. This is worsened by the challenge of the availability land.

Biochemical oxygen demand (BOD) is an indicator used to evaluate the degree of the contamination of water and sewages with the organic substances [12]. The aim of this study is therefore to develop, calibrate and verify a model for the prediction of biochemical oxygen demand in integrated solar and hydraulic jump enhanced waste stabilization pond.

2. MATERIALS AND METHODS

2.1. Area of Study

The study was carried out at the University of Nigeria, Nsukka. The geographical location of the experimental pond was approximately at 7.4°E, 6.9°N. The sewage treatment plant consists of a screen followed by two imhoff tanks and two facultative waste stabilization ponds. Sludge discarded from the imhoff tank is placed in the drying beds. The efficiency of the sewage treatment has deteriorated due to population growth. However, its effluent is used for uncontrolled vegetable irrigation by some village dwellers.
2.2. Description of Experimental Setup
Three sets of experimental pond with varying locations of change in pond bed slope were constructed using metallic tanks. The pond was constructed with tilt frames of size 1.0m x 0.3m, fixed at varying angles in accordance with the relative position of the sun per week (Figure 1). The tilt frame was made of flat wooden board wrapped with aluminum foil paper to serve as solar reflectors (west facing). Half-inch diameter inlet and outlet pipes were fitted centrally to the experimental ponds. To control the inflow and outflow, valves were fitted at the inlet and outlet pipes of the experimental ponds. Two storage tanks were usually filled to supply the pond with sewage effluent from the imhoff tank of the University of Nigeria, Nsukka sewage treatment plant through a hose with the aid of an electromechanical water pump. The influent samples for the laboratory analysis were obtained from the storage tank immediately after being filled. Also, the experimental ponds were immediately filled and samples collected at the outlets after two days.

![Figure 1: Schematic diagram of experimental setup](image)

2.3. Data Collection and Laboratory Analysis
Wastewater samples were collected before degradation in the ISHJEWSP. Also, treated wastewater samples were collected after degradation in the ISHJEWSP. The effluent samples were collected for varying inlet velocities and varying locations of point of initiation of hydraulic jump. The samples were examined for physicochemical and biological characteristics for a period of nine months. The parameters examined were temperature, pH, detention time, dissolved oxygen (DO), total coliform count (TCC), total suspended solids (TSS), E-coli, algae concentration and biochemical oxygen demand (BOD). All the laboratory analyses were carried out using appropriate water testing meters and in accordance with the standard methods [13].

2.5. Model Development
In developing the model, it was assumed that a relationship existed between the effluent biochemical oxygen demand concentration \(N_e\) in the integrated solar and hydraulic jump enhanced waste stabilization pond and the influent concentration of \(N_o\), intensity of solar radiation \(I\), influent algae concentration \(A\), inlet
velocity \((V)\), density of wastewater \((\rho)\), characteristic length of the pond \((L)\), detention time \((t)\), dispersion coefficient \((\varepsilon)\) and dissolved oxygen concentration \((DO)\).

Applying the MLT dimensional analysis approach using Buckingham’s \(\pi\)-theorem, we have

\[
N_e = f(N_o, I, A, V, \rho, L, t, DO)
\]

\[
f_1(N_e, N_o, I, A, V, \rho, L, t, DO) = 0
\]  

(1)

Total number of variables = 10

Writing the dimensions of each variable, we get

\[
N_e = ML^{-3}, \quad N_o = ML^{-3}, \quad I = MT^{-3}, \quad V = L \cdot T^{-1}, \quad \rho = ML^{-3}, \quad L = L, \quad t = T, \quad A = ML^{-3}, \quad DO = ML^{-3}
\]

The number of fundamental dimensions, \(m = 3\), Number of \(\pi - \) terms = 10 – 3 = 7

Equation (2) can be written as

\[
f_1(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) = 0
\]  

(3)

The parameters of length, velocity, and density were selected as repeating variables corresponding to geometric, flow and fluid properties, respectively. Substituting the solutions of the \(\pi - \) terms i.e \(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7\) in Equation (3), we get

\[
f_1\left(\frac{N_e}{\rho}, \frac{N_o}{\rho} \cdot \frac{l}{\rho V^3}, \frac{A}{\rho}, \frac{\varepsilon}{L V}, \frac{V}{L}, \frac{DO}{\rho}\right) = 0
\]

\[
\therefore \frac{N_e}{\rho} = \emptyset \left[\frac{N_o}{\rho} \cdot \frac{l}{\rho V^3}, \frac{A}{\rho}, \frac{\varepsilon}{L V}, \frac{V}{L}, \frac{DO}{\rho}\right]
\]  

(4)

(5)

It was assumed that a non-linear relationship exists between the response variable and predictor variables. Therefore, Equation (5) can be written as;

\[
\frac{N_e}{\rho} = \left(\frac{N_o}{\rho}\right)^{a_1} \left(\frac{l}{\rho V^3}\right)^{a_2} \left(\frac{A}{\rho}\right)^{a_3} \left(\frac{\varepsilon}{L V}\right)^{a_4} \left(\frac{V}{L}\right)^{a_5} \left(\frac{DO}{\rho}\right)^{a_6}
\]  

(6)

Taking logarithm of both sides, we get

\[
\ln\left(\frac{N_e}{\rho}\right) = \ln\left(\left(\frac{N_o}{\rho}\right)^{a_1} \left(\frac{l}{\rho V^3}\right)^{a_2} \left(\frac{A}{\rho}\right)^{a_3} \left(\frac{\varepsilon}{L V}\right)^{a_4} \left(\frac{V}{L}\right)^{a_5} \left(\frac{DO}{\rho}\right)^{a_6}\right)
\]

\[
\ln\left(\frac{N_e}{\rho}\right) = a_1 \ln\left(\frac{N_o}{\rho}\right) + a_2 \ln\left(\frac{l}{\rho V^3}\right) + a_3 \ln\left(\frac{A}{\rho}\right) + a_4 \ln\left(\frac{\varepsilon}{L V}\right) + a_5 \ln\left(\frac{V}{L}\right) + a_6 \ln\left(\frac{DO}{\rho}\right)
\]  

(7)

Linearizing Equation (7), we get

\[
\ln\left(\frac{N_e}{\rho}\right) = a_1 \ln\left(\frac{N_o}{\rho}\right) + a_2 \ln\left(\frac{l}{\rho V^3}\right) + a_3 \ln\left(\frac{A}{\rho}\right) + a_4 \ln\left(\frac{\varepsilon}{L V}\right) + a_5 \ln\left(\frac{V}{L}\right) + a_6 \ln\left(\frac{DO}{\rho}\right)
\]  

(8)

Equation (8) is similar to Equation (9)

\[
Y = a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_4 + a_5 x_5 + a_6 x_6 + a_7 x_7
\]  

(9)

Where \(y \equiv \ln\left(\frac{N_e}{\rho}\right), \quad x_1 \equiv \ln\left(\frac{N_o}{\rho}\right), \quad x_2 \equiv \ln\left(\frac{l}{\rho V^3}\right), \quad x_3 \equiv \ln\left(\frac{A}{\rho}\right), \quad x_4 \equiv \ln\left(\frac{\varepsilon}{L V}\right), \quad x_5 \equiv \ln\left(\frac{V}{L}\right), \quad x_6 \equiv \ln\left(\frac{DO}{\rho}\right)
\]

\[2.5. \text{Model Calibration}\]

If the least square form of Equation (9) is minimized, then, the following equations are obtained.

\[
\Sigma y x_1 = a_1 \Sigma x_1^2 + a_2 \Sigma x_1 x_2 + a_3 \Sigma x_1 x_3 + x_4 \Sigma x_1 x_4 + a_5 \Sigma x_1 x_5 + a_6 \Sigma x_1 x_6
\]

\[
\Sigma y x_2 = a_1 \Sigma x_1 x_2 + a_2 \Sigma x_2^2 + a_3 \Sigma x_2 x_3 + a_4 \Sigma x_2 x_4 + a_5 \Sigma x_2 x_5 + a_6 \Sigma x_2 x_6
\]  

(10)

(11)
Dimensional Analysis Design Model of Biochemical Oxygen Demand In Integrated Solar and Hydraulic Jump Enhanced Waste Stabilization Pond

\[ \sum y_3 = a_1 \sum x_1 x_3 + a_2 \sum x_2 x_3 + a_3 \sum x_3^2 + a_4 \sum x_3 x_4 + a_5 \sum x_3 x_5 + a_6 \sum x_3 x_6 \]  \quad (12)

\[ \sum y_4 = a_1 \sum x_1 x_4 + a_2 \sum x_2 x_4 + a_3 \sum x_3 x_4 + a_4 \sum x_4^2 + a_5 \sum x_4 x_5 + a_6 \sum x_4 x_6 \]  \quad (13)

\[ \sum y_5 = a_1 \sum x_1 x_5 + a_2 \sum x_2 x_5 + a_3 \sum x_3 x_5 + a_4 \sum x_4 x_5 + a_5 \sum x_5^2 + a_6 \sum x_5 x_6 \]  \quad (14)

\[ \sum y_6 = a_1 \sum x_1 x_6 + a_2 \sum x_2 x_6 + a_3 \sum x_3 x_6 + a_4 \sum x_4 x_5 + a_5 \sum x_5 x_6 + a_6 \sum x_6^2 \]  \quad (15)

A total of 108 data sets were obtained from the study. The model was calibrated with the first 54 data set obtained from the experimental ISHJEWSP. The parameters were factored in accordance with the terms in Equations (10) to (15). Substituting appropriate terms in Equations (10) to (15), Equation (16) was obtained in matrix form.

\[
\begin{bmatrix}
1702.87 & 513.15 & -2811.37 & -449.65 & 3179.39 & -1865.91 \\
513.15 & 180.86 & -842.85 & -134.42 & 954.89 & -564.17 \\
-2811.37 & -842.85 & 4644.42 & 742.82 & -5251.39 & 3081.37 \\
-449.65 & -134.42 & 742.82 & 119.14 & -839.81 & 492.93 \\
3179.39 & 954.89 & -5251.39 & -839.81 & 5938.45 & -3484.67 \\
-1865.91 & -564.17 & 3081.37 & 492.93 & -3484.67 & 2046.26 \\
\end{bmatrix}
\begin{bmatrix}
 a_1 \\
 a_2 \\
 a_3 \\
 a_4 \\
 a_5 \\
 a_6 \\
\end{bmatrix}
=
 \begin{bmatrix}
1178.32 \\
348.05 \\
-1946.78 \\
-311.45 \\
2201.19 \\
-1289.60 \\
\end{bmatrix}
\]  \quad (16)

Equation (16) is represented in a compact matrix form as Equation (17):

\[
\{A\} = [X]^{-1}\{B\}^{Transpose}
\]  \quad (17)

Solving Equation (17) using MATLAB v.7.7.47 (R2008b) yields the following results for \( a_1, a_2, a_3, a_4, a_5, a_6 \) were 0.6142, -0.0276, 0.0615, -0.9037, 0.8667, 1.5234, respectively.

Therefore, Equation (6) becomes

\[
N_e = \rho \left[ \left( \frac{N_o}{\rho} \right)^{0.6142} \left( \frac{V}{\rho^2} \right)^{-0.0276} \left( \frac{A}{\rho} \right)^{0.0615} \left( \frac{\varepsilon}{L} \right)^{-0.9037} \left( \frac{DO}{L} \right)^{0.8667} \left( \frac{DO}{L} \right)^{1.5234} \right]^{0.037}
\]  \quad (18)

In a compact form, Equation (18) becomes:

\[
N_e = \frac{N_o^{0.6142} V^{1.8532} DO^{1.5234} A^{0.0615} L^{0.037}}{\rho^{1.1715} \varepsilon^{0.9037} I^{0.0276}}
\]  \quad (19)

Considering the change in slope causing the occurrence of the hydraulic jump, we have the length of channel \( L \) to be

\[
L = \frac{h}{\sin \alpha} + x
\]  \quad (20)

Substituting Equation (20) in Equation (19), we get

\[
N_e = \frac{N_o^{0.6142} V^{1.8532} DO^{1.5234} A^{0.0615} \left( \frac{h}{\sin \alpha} + x \right)^{0.037}}{\rho^{1.1715} \varepsilon^{0.9037} I^{0.0276}}
\]  \quad (21)

Where: \( N_e \) is the effluent biochemical oxygen demand concentration in the ISHJEWSP (g/m\(^3\)), \( N_o \) is the influent concentration of BOD (g/m\(^3\)), I is the intensity of solar radiation (W/m\(^2\)), A is the influent algae concentration (g/m\(^3\)), V is the inlet velocity (m/s), \( \rho \) is the density of wastewater (g/m\(^3\)), L is the characteristic length of the pond (m), t is the detention time (s), \( \varepsilon \) is the dispersion coefficient (m\(^2\)/s), DO is the concentration of dissolved oxygen (g/m\(^3\)), X is the length of the horizontal section of the pond (m), \( \alpha \) is the angle denoting change in pond bed slope (\(^\circ\))
2.6. Statistical Evaluation of Model

The performance indicator parameters of the developed model included the coefficient of correlation and the coefficient of determination. In the past, standard theories of regression analysis have been discussed [14-18]. The performance indicators are shown in Equations (22) and (23).

The correlation coefficient between \( Y \) and \( \hat{Y} \), is given by:

\[
Cor(Y, \hat{Y}) = \frac{\sum(y_i - \bar{y}) \sum(\hat{y}_i - \bar{\hat{y}})}{\sqrt{\sum(y_i - \bar{y})^2 \sum(\hat{y}_i - \bar{\hat{y}})^2}}
\]

(22)

The coefficient of determination is also given as

\[
R^2 = [Cor(Y, \hat{Y})]^2
\]

(23)

Where \( y_i \) is the ith value of the measured variable \( Y \)
- \( \bar{y} \) is the mean of the measured variable \( Y \)
- \( \hat{y}_i \) is the ith value of the calculated variable \( \hat{Y} \)
- \( \bar{\hat{y}} \) is the mean of the calculated variable \( \hat{Y} \)

3. RESULTS AND DISCUSSION

Equation (21) is obtained as the non-linear dimensional analysis design model for the prediction of biochemical oxygen demand in integrated solar and hydraulic jump enhanced waste stabilization pond with respect to the University of Nigeria, Nsukka treatment plant. The model was calibrated with the first 54 data set and due to paucity of empirical literatures, was verified with the remaining 54 data from the experimental ISHJEWSP (see Figure 2).

The verification of the model gave a good coefficient of correlation of \( R = 0.864 \) between the measured and calculated concentration of effluent BOD. At a significance level of 0.05, the regressed slope, intercept and standard error are 0.938, 5.564 and 8.162 respectively.

The predicted values of BOD were close to the experimental values. The coefficient of determination (R squared) shows that 74.68% of the variation of the experimental results is explained by the developed model. Therefore, the prediction of BOD concentration in the ISHJEWSP would be possible from the developed model as the coefficient of correlation of 0.851 is adequate (see Table 1).
Dimensional Analysis Design Model of Biochemical Oxygen Demand In Integrated Solar and Hydraulic Jump Enhanced Waste Stabilization Pond

Figure 2 Measured $N_e$ versus calculated $N_e$ in ISHJEWSP

Table 1 The summary of output of linear regression analysis between the measured and calculated Effluent BOD in the ISHJEWSP

<table>
<thead>
<tr>
<th>Regression Statistics</th>
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<tbody>
<tr>
<td>Multiple R</td>
<td>0.864174278</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>R Square</td>
<td>0.746797184</td>
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<tr>
<td>Adjusted R Square</td>
<td>0.741927899</td>
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<tr>
<td>Standard Error</td>
<td>8.162048249</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>54</td>
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<td>MS</td>
<td>F</td>
<td>Significance F</td>
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<td>10217.29</td>
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<td>Residual</td>
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<td>3464.19</td>
<td>66.61903</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>53</td>
<td>13681.48</td>
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<tr>
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<th>t Stat</th>
<th>P-value</th>
<th>Lower 95%</th>
<th>Upper 95%</th>
<th>Lower 95.0%</th>
<th>Upper 95.0%</th>
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<tr>
<td>Intercept</td>
<td>5.564418279</td>
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<td>1.596371</td>
<td>-1.43009</td>
<td>12.55892</td>
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<td>12.55892</td>
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<tr>
<td>X Variable 1</td>
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<td>0.075707</td>
<td>12.38422</td>
<td>3.92E-17</td>
<td>0.785659</td>
<td>0.785659</td>
<td>1.089496</td>
</tr>
</tbody>
</table>

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4. CONCLUSIONS
A design model for the prediction of biochemical oxygen demand (BOD) in the integrated solar and hydraulic jump enhanced waste stabilization pond was derived based on dimensional analysis using the Buckingham’s $\pi$-theorem approach. The correlation coefficient, regressed slope, intercept and standard error obtained were 0.864, 0.938, 5.564 and 8.162 respectively at $\alpha = 5\%$ level of significance. The coefficient of determination (R square) showed that 74.68% of the variation of the experimental results is explained by the developed model. Therefore, the prediction of BOD concentration in the ISHJEWSP would be possible from the developed model.

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