IMPROVING THE STRUT AND TIE METHOD BY INCLUDING THE CONCRETE SOFTENING EFFECT

Mohammed Mohammed Rasheed
Associated Professor in Civil Engineering Department, College of Engineering.
Al-Mustansiriyah University, Baghdad, Iraq.

Sawsan Akram Hassan
Associated Professor in Civil Engineering Department.
Al-Mustansiriyah University, Baghdad, Iraq.

Sara Saad Faraj
M.Sc. Student in Civil Engineering Department.
Al-Mustansiriyah University, Baghdad, Iraq.

ABSTRACT

Strut and tie model approach evolves as one of the most useful analysis methods for shear critical structures and for other disturbed regions in concrete structures. The main objective of this research is modified the strut and tie method. As especially of the softening strut and tie method, the following work concern for determining a new factor (named-correction factor \( \lambda \)), and the formulation is based on the failure criterion from the Mohr Coulomb Theory for nodal zones (tension-compression stress state). To support the theoretical work eleven simply supported reinforced concrete deep beams have been casting and testing. The parameters considered in this study are; shear span to total depth \((a/h)\), compressive strength of concrete \((f_c)\) vertical web reinforcement \((p_v)\) and horizontal web reinforcement \((p_h)\). The effect of these variables on the; cracking load, ultimate load, load-midspan deflection response, crack pattern, and strains of concrete and steel reinforcement have been studied. According to the experimental results, a relationship between the correction factor \( \lambda \) and the variables (shear span to total depth ratio, concrete compressive strength, vertical web reinforcement and horizontal web reinforcement) was assumed.
1. INTRODUCTION
To define the theoretical behavior of beam, in general, the behavior of concrete beam can be divided into two general regions namely; the main regions or flexural regions (Bernoulli or B-regions) and the local regions or regions near discontinuities (Disturbed or D-regions). The local regions include the ends of a beam, the connections between a beam and a column, the corbels, the region adjacent to a concentrated load, etc. The areas include the primary portions of each member away from the local regions are called the main regions [1].

In B-regions, the stresses and strains are distributed so regularly that they can easily be expressed mathematically. That is, the stresses and strains in the main regions are governed by simple equilibrium and compatibility conditions. In the case where beams are subjected to shear, the stresses and strains should satisfy the two-dimensional equilibrium and compatibility conditions [1].

In D-regions, the stresses and strains are so disturbed and irregular that they are not amenable to mathematical solution. In particular, the compatibility conditions are difficult to apply. In the design of D-regions the stresses are usually determined by equilibrium condition alone, while the strain conditions are neglected. Numerical analysis by computer (such as the finite element method), can possibly determine the stress and strain distributions in the local regions, but it is seldom employed due to its complexity [1].

The truss model had been applied to treat shear of reinforced concrete members since the turn of the 20th century [2].

A reinforced concrete element subjected to shear stresses is actually subjected to biaxial stresses (principal compression and principal tension) so, the shear action defined as a two-dimensional problem. However, the prediction of shear strengths based on truss model consistently overestimated that of tested specimens.

The mistake in applying the truss model theory before 1972 was the use of the compressive stress-strain relationship of concrete obtained from the uniaxial tests of standard cylinders without considering the two-dimensional effect [3].

2. MODIFIED SOFTENED STRUT AND TIE MODEL
A modified softened strut and tie model (MSST) derived for determining the shear strength of reinforced concrete deep beams. The method is based on Mohr Coulomb failure criterion. The concrete softening effect and the stress distribution factor, $K$, based on force and moment equilibrium satisfaction.

The model utilizes a failure criterion from the modified Mohr Coulomb theory for nodal zones (tension-compression stress state) as below:
Improving the Strut and Tie Method by Including the Concrete Softening Effect

\[ \frac{f_1}{f_t} + \frac{f_2}{f_c} = 1 \] (1)

Where \( f_1 \) and \( f_2 \) are principal tensile and compressive stresses at the nodal zone respectively, \( f_c \) is the concrete compressive strength in the \( f_2 \) direction and \( f_t \) represents the maximum combined tensile strength of both reinforcement and concrete in \( f_1 \) direction.

According to Wang and Meng [4] the equation modified to:

\[ \frac{f_1}{f_t} + \lambda \frac{f_2}{f_c} = 1 \] (2)

They assumed \( \lambda = 0.8 \). Here we assume \( \lambda \) as a function of many variables.

3. DERIVATION OF SHEAR STRENGTH

A. Bottom Nodal Zone

From the equilibrium of forces at the bottom nodal zone of the inclined strut, the following equations can be obtained, as shown in "Fig. 1":

\[ F_c = \frac{V_n}{\sin \theta_s} \] (3)

\[ T_s = \frac{V_n}{\tan \theta_s} \] (4)

Where \( F_c \) and \( T_s \) are the forces in the primary strut and bottom tension tie, respectively. \( V_n \) is the shear strength of the beam. The inclined angle of the primary strut \( \theta_s \) can be computed from:

\[ \tan \theta_s = \frac{h - \frac{L_d}{2} + \frac{L_c}{2}}{\frac{a}{2}} \] (5)

where \( h \) is the total deep beam depth, \( d \) is the effective depth of deep beam, \( L_c \) and \( L_d \) are the depths of bottom and top nodal zones, respectively.

\[ L_c = 2(h - d) \] (6)
and a is the shear span measured from center lines of the load and support bearing plates. The term \( L_d \) is initial unknown. For convenience and simplicity, assuming \( L_d = L_c \) gives an error less than 2\% due to that \( L_d \) is typically ten times smaller than the total deep beam depth \( h \), [5, and 6].

The principal tensile stress \( f_1 \) at the bottom nodal zone arises from the component force of longitudinal reinforcement in the direction perpendicular to the diagonal strut, as shown in "Fig. 2", namely, \( T_s \sin \theta_s \) as follows:

\[
f_1 = \frac{K T_s \sin \theta_s}{A_c/ \sin \theta_s} = K P_t
\]

where \( P_t \) is the average equivalent tensile stress across the diagonal strut and \( A_c \) is the effective cross sectional area of the beam \( (A_c = b_w d_c) \). \( K \) is a factor taking in to account of the non uniformity of the stress distribution. \( A_s \) shown in "Figure. 2", considering one reinforcing bar that inclines at an angle \( \theta_w \) from horizontal. From force equilibrium in the \( f_1 \) direction, the following equation can be established:

\[
\left(\frac{K+K'}{2}\right) P_t b_w d_c/ \sin \theta_s = T \sin(\theta_s + \theta_w)
\]

From moment equilibrium about the top node, gives:

\[
\left(\frac{K-K}{3}\right) (d_c/ \sin \theta_s)^2 b_w P_t = T \sin(\theta_s + \theta_w) \frac{d_w}{\sin \theta_s}
\]

From Eqs. (8) and (9), the factors \( K \) and \( K' \) can be obtained

\[
K = 4 - 6 \frac{d_w}{d_c}
\]

\[
K' = 6 \frac{d_w}{d_c} - 2
\]

**Figure 2.** Assumed stress distribution due to one bar.

For the case of bottom reinforcement, \( d_w = d_c \), \( \theta_w = 0 \), the stress distribution factors

\[
K = -2 \text{ and } K = 4
\]
Improving the Strut and Tie Method by Including the Concrete Softening Effect

For web reinforcement, assume that there are \( n_s \) web steel bars evenly distributed along the strut, the stress distribution factors can be written as below:

\[
\hat{K} = \sum_{i=1}^{n_s} 4 - 6 \frac{d_{wi}}{d_c} = n_s \quad (11a)
\]

\[
K = \sum_{i=1}^{n_s} 6 \frac{d_{wi}}{d_c} - 2 = n_s \quad (11b)
\]

In a similar fashion as Eq. (7), the tensile capacity, \( f_t \), at the bottom nodal zone can be expressed as follows:

\[
f_t = f_{st} + f_{ct} \quad (12)
\]

where \( f_{st} \) represents the contribution from steel reinforcement, as follows:

\[
f_{st} = f_{ss} + f_{sw} \quad (13)
\]

\( f_{ss} \) represents the contribution of bottom longitudinal steel, and can be calculated as:

\[
f_{ss} = \frac{4A_{sw}f_y \sin \theta_s}{A_c / \sin \theta_s} \quad (14)
\]

\( f_{sw} \) represents the contribution of web reinforcement at the interface of nodal zone, and can be calculated as:

\[
f_{sw} = \frac{A_{svf_y \sin (\theta_s + \theta_w)}}{A_c / \sin \theta_s} \quad (15)
\]

\[
A_{sw} = nA_{sw1} \quad (16)
\]

\( A_{sw} \) represents the total area of web reinforcement crossing the concrete strut. For general case of vertical and horizontal web reinforcement, Eq. (15) can be written as below:

\[
f_{sw} = \frac{A_{svf_y \sin 2\theta_s}}{2A_c} + \frac{A_{shf_y \sin 2\theta_s}}{A_c} \quad (17)
\]

\( A_{sv} \) and \( A_{sh} \) are the total areas of vertical and horizontal web reinforcement, respectively.

The concrete tensile strength of crack reinforced concrete proposed by Belarbi and Hsu [7] can be calculated from the following relation:

\[
f_{ct} = 0.31 \sqrt{\frac{E_t}{\varepsilon_{tr}}} \left( \frac{f_{ct}}{E_t} \right)^{0.4} \quad (18)
\]

where \( \varepsilon_{tr} \) is the strain of concrete at cracking, \( \varepsilon_1 \) is the principle tensile strain of concrete strut. So, Eq. (12) can be rearranged as bellow:

\[
f_t = \frac{4A_{sw}f_y \sin \theta_s}{A_c} + \frac{A_{swf_y \sin 2\theta_s}}{2A_c} + \frac{A_{shf_y \sin 2\theta_s}}{A_c} + f_{ct} \quad (19)
\]

The principal compressive stress \( f_2 \) in the direction of the strut at the bottom nodal zone can be computed by:

\[
f_2 = \frac{f_e - T \cos \theta_s}{A_{str}} \quad (20)
\]

where \( A_{str} \) is the cross sectional area of strut at the bottom nodal zone and is defined as following:

\[
A_{str} = b_w(l_c \cos \theta_s + l_h \sin \theta_s) \quad (21)
\]

Substituting Eq. (4) in Eq. (7) gives:
Substituting Eqs. (3), (4) in Eq. (20) give:

\[ f_1 = \frac{4V_n \sin \theta_s \cos \theta_s}{A_c} \]  

(22)

Substituting Eqs. (22) and (23) in Eq. (2) give:

\[ V_n = \frac{1}{A_c f_t} \left( \frac{\sin \theta_s \cos \theta_s}{\lambda \sin \theta_s} + \frac{\lambda \sin \theta_s}{\lambda \sin \theta_s} \right) \]

\[ \lambda = \frac{\frac{f_c A_{str}}{V_n \sin \theta_s} - \frac{4 \cos \theta_s f_c A_{str}}{A_c f_t}}{2} \]

(24)

(25)

B. Top Nodal Zone

The top nodal is subjected to a biaxial compression-compression stress state, the failure mode is:

\[ \lambda f_c^2 = 1 \]  

(26)

So, substitute Eq. (23) into Eq. (26) gives:

\[ V_n = \frac{f_c A_{str}}{2 \lambda \sin \theta_s} \]

\[ \lambda = \frac{f_c A_{str}}{2 V_n \sin \theta_s} \]

(27)

(28)

4. EXPERIMENTAL INVESTIGATION

The experimental program was planned and executed to investigate the effect of the correction factor and the shear strength for simply supported reinforced concrete deep beams. The experimental program consisted of testing eleven simply supported deep beams, which have an overall length of 1200 mm, width of 115 mm and overall depth which varied 333 mm, 400 mm and 500 mm. All beams have the same flexural reinforcement (2 Ø 20). The parameters considered in this study are the shear span to total depth ratio \((a/h)\), the compressive strength of concrete \((f'c)\) and the amounts of vertical and horizontal shear reinforcements \((\rho_v, \rho_h)\). The beams loaded identically by vertical concentrated load acting at the top of the beams at mid span, as shown in "Fig. 3". Precautions were made to avoid flexural failure at mid span and local failure at load and support points by means of steel reinforcement and steel plates, respectively. Details of the eleven tested reinforced concrete deep beams are listed in Table (1). The test specimens were designed according to ACI318M-11 by using the strut and tie model [8].

5. EXPERIMENTAL DATA

This section includes the important variables that affect the capacity and behavior of deep beams with good ranges for values of these variables. These variables and their ranges are as follows:

- Shear span to total depth ratio \((a/h)\) which ranges from 0.8 to 1.2.
- Concrete compressive strength \((f'c)\) which ranges between 23.2 MPa and 39.89 MPa.
- Vertical web reinforcement \((\rho_v)\) which ranges between 0 and 0.005.
- Horizontal web reinforcement \((\rho_h)\) which ranges between 0 and 0.005.
Table 1. Details of Tested Beams

<table>
<thead>
<tr>
<th>Spec. No.</th>
<th>h (mm)</th>
<th>$F'_c$ (MPa)</th>
<th>Web R. spacing (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Vertical</td>
</tr>
<tr>
<td>DB1</td>
<td>400</td>
<td>34.8</td>
<td>Ø 4 @ 200</td>
</tr>
<tr>
<td>DB2</td>
<td>400</td>
<td>33.7</td>
<td>-</td>
</tr>
<tr>
<td>DB3</td>
<td>400</td>
<td>34.8</td>
<td>Ø 6 @ 160</td>
</tr>
<tr>
<td>DB4</td>
<td>400</td>
<td>34.8</td>
<td>-</td>
</tr>
<tr>
<td>DB5</td>
<td>400</td>
<td>33.7</td>
<td>Ø 6 @ 80</td>
</tr>
<tr>
<td>DB6</td>
<td>400</td>
<td>33.7</td>
<td>-</td>
</tr>
<tr>
<td>DB7</td>
<td>400</td>
<td>34.2</td>
<td>Ø 4 @ 60</td>
</tr>
<tr>
<td>DB8</td>
<td>500</td>
<td>34.2</td>
<td>Ø 4 @ 130</td>
</tr>
<tr>
<td>DB9</td>
<td>400</td>
<td>23.2</td>
<td>Ø 4 @ 200</td>
</tr>
<tr>
<td>DB10</td>
<td>333</td>
<td>34.2</td>
<td>Ø 4 @ 265</td>
</tr>
<tr>
<td>DB11</td>
<td>400</td>
<td>39.9</td>
<td>Ø 4 @ 200</td>
</tr>
</tbody>
</table>

6. SPECIMENS BEHAVIOR AND FAILURE MODE

In general, a few flexural cracks formed first, which remained at a narrow width throughout the tests. Diagonal cracks then formed at approximately 25% of the peak load, as mentioned earlier, defining the direction of the main concrete strut.

Failure for all test specimens was brittle and the failure modes were identified as follows:

- Diagonal-splitting failure, in which diagonal cracks that formed initially at mid-depth of the beam in the direction of the main strut propagated to the outside edge of the loading plate and the inside edge of the bearing plate at the support, as appear in specimens DB7 & DB11 (Figure 4).

- Strut crushing failure at beam mid-depth following the formation of several diagonal cracks, as appear in specimens DB3, DB5, DB8 & DB10 (Figure 4).

Figure 3. Details of the tested beam (control beam) (All dimensions are in mm).
Mohammed Mohammed Rasheed, Sawsan Akram Hassan and Sara Saad Faraj

- Shear-compression failure near the loading point after formation of one or two main diagonal cracks, which had developed at mid-depth of the beam and propagated toward the outside edge of the loading plate, as appear in specimens DB1, DB2, DB4, DB6 & DB9 (Figure 4).

7. THE PROPOSED EQUATION FOR SHEAR STRENGTH

The proposed equation for predicted shear strength \( V_n \) without including correction factor defined as following:

\[
V_n = \frac{1}{\frac{4 \sin \theta_c \cos \theta_c \sin \theta_c}{Ac} \cdot \frac{1}{\lambda \sin \theta_c \cdot \frac{Ac}{ft}}}
\]

The proposed equation for predicted shear strength \( V_n \) including correction factor will be defined as following:

\[
V_n = \frac{1}{\frac{4 \sin \theta_c \cos \theta_c \sin \theta_c}{Ac} \cdot \lambda \sin \theta_c \cdot \frac{1}{\frac{Ac}{ft}}}
\]

where \( \lambda \) is assumed as correction factor. It is assumed as a function of the variables.
Improving the Strut and Tie Method by Including the Concrete Softening Effect

8. CORRECTION FACTOR ($\lambda$)

The proposed formula of correction factor will consist of four terms. The first term depends on material properties (compressive strength of concrete). The second term depends on beam geometry (shear span to total depth). The third and fourth terms depend on web reinforcement values, as follow:

$$\lambda = f_1(f'_c)\cdot f_2(a/h)\cdot f_3(\rho_v)\cdot f_4(\rho_h)$$  \hspace{1cm} (31)

where:

$$f_1 = -1500 + 170 f'_c - 3 f'_c^2$$  \hspace{1cm} (32)

$$f_2 = -0.075 + 0.2 (a/h) - 0.105 (a/h)^2$$  \hspace{1cm} (33)

$$f_3 = 0.4 - 190 \rho_v + 25400 \rho_v^2$$  \hspace{1cm} (34)

$$f_4 = 0.65 - 105 \rho_h + 11880 \rho_h^2$$  \hspace{1cm} (35)

where $\lambda$ equal to one when $f'_c > 30$MPa, $\rho_w \geq 0.0025$ and $\rho_v \geq 0.0006$.

The coefficients of this proposed formula are determined by regression analysis. These formulas are based on the test results of 25 deep beams (experimental data of

Figure 4. Crack pattern for deep beam specimens after testing.
eleven reinforced concrete deep beams of this study and the experimental data from Hassan [9]). "Fig. 5" shows the relation between assumed correction factor (Eq.31) and experimental results (Eq.25).

The results of the equation of shear strength with and without correction factor are listed in Table (2), the average value for experimental to predict shear strengths is 0.989, the standard deviation value for experimental to predict shear strengths is 0.061 and the coefficient of multiple determination ($R^2$) value is 0.929.

<table>
<thead>
<tr>
<th>Spec. No.</th>
<th>$V_n$ (exp.) (kN)</th>
<th>Without correction factor</th>
<th>With correction factor</th>
<th>$V_n$ (Eq.29) (kN)</th>
<th>$V_n$ (Eq.30) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_n$ (Eq.29)</td>
<td>$V_n$ (exp.) / $V_n$ (Eq.29)</td>
<td>$\lambda$</td>
<td>$V_n$ (Eq.30)</td>
<td>$V_n$ (exp.) / $V_n$ (Eq.30)</td>
</tr>
<tr>
<td>DB1</td>
<td>199</td>
<td>199.6</td>
<td>0.997</td>
<td>1</td>
<td>199.6</td>
</tr>
<tr>
<td>DB2</td>
<td>94.3</td>
<td>186.9</td>
<td>0.504</td>
<td>4.28</td>
<td>89.6</td>
</tr>
<tr>
<td>DB3</td>
<td>217.5</td>
<td>198.5</td>
<td>1.096</td>
<td>0.65</td>
<td>225.6</td>
</tr>
<tr>
<td>DB4</td>
<td>134.5</td>
<td>203.9</td>
<td>0.66</td>
<td>2.64</td>
<td>129.5</td>
</tr>
<tr>
<td>DB5</td>
<td>209</td>
<td>197.2</td>
<td>1.06</td>
<td>0.91</td>
<td>203.7</td>
</tr>
<tr>
<td>DB6</td>
<td>99</td>
<td>194.3</td>
<td>0.509</td>
<td>2.91</td>
<td>117.2</td>
</tr>
<tr>
<td>DB7</td>
<td>207.5</td>
<td>193.3</td>
<td>1.073</td>
<td>0.61</td>
<td>222.3</td>
</tr>
<tr>
<td>DB8</td>
<td>208.5</td>
<td>217.2</td>
<td>0.96</td>
<td>1</td>
<td>217.2</td>
</tr>
<tr>
<td>DB9</td>
<td>119</td>
<td>167.8</td>
<td>0.709</td>
<td>2.18</td>
<td>111</td>
</tr>
<tr>
<td>DB10</td>
<td>178.5</td>
<td>180</td>
<td>0.992</td>
<td>1</td>
<td>180</td>
</tr>
<tr>
<td>DB11</td>
<td>204</td>
<td>204.6</td>
<td>0.997</td>
<td>1</td>
<td>204.6</td>
</tr>
<tr>
<td>Avg.</td>
<td></td>
<td>0.869</td>
<td></td>
<td></td>
<td>0.989</td>
</tr>
<tr>
<td>S.D</td>
<td></td>
<td>0.217</td>
<td></td>
<td></td>
<td>0.061</td>
</tr>
</tbody>
</table>

**Table 2. Results of Experimental and Theoretical Shear Strengths**

**Figure 5.** Experimental and proposed correction factor.
9. CONCLUSIONS
The mistake in applying the truss model methods were the use of the compressive stress-strain relationship of concrete obtained from the uniaxial tests of standard cylinders without considering the two-dimensional effect. Every analysis method has advantages and disadvantages, and the most disadvantage of the strut and tie method may be the neglected the softening effect. So, the purpose of this study is concern for determining a new factor (named-correction factor \( \lambda \)) and the derivation of an equation for this factor including the effect of variables effect in it.

The relationship between the correction factor and the variables was assumed. The proposed equation gives acceptable agreement with the experimental results. The average value for experimental to predict shear strengths is 0.989, the standard deviation value for experimental to predict shear strengths is 0.061 and the coefficient of multiple determination is 0.929. This conclusion confirms the accuracy and rationality of this proposed equation.

REFERENCES


[8] ACI Committee 318, Building Code Requirements for Structural Concrete, (ACI 318M-11) and commentary (318R-11), American Concrete Institute, Farmington Hills, Michigan, USA, 2011.