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SUGGESTING DEFLECTION EXPRESSIONS FOR RC BEAMS

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ABSTRACT

This experimental work aims at presenting load-deflection expressions for the concrete beams that reinforced with three different reinforcement ratios of ACI 318-14, which are minimum, maximum and the average of them. Three groups of beams were cast, each group contained three beam specimens. Three types of loading are used, 1-concentrated force, 2-concentrated forces and partial uniformly distributed load. It is also seen that, when reinforcing ratio increases from minimum to maximum, in case of 1-concentrated force, ultimate capacity increases by about 280% and deflection decreases by about 33%, respectively. Whereas, in case of 2-concentrated forces, ultimate capacity increases by about 258% and deflection decreases by about 50%, respectively. Finally, in case of uniformly distributed load, ultimate capacity increases by about 289% and deflection decreases by about 28%, respectively.

Keywords: Beam, Deflection, RC, simply supported, ultimate capacity, expression, reinforcement ratio.

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1. INTRODUCTION

Reinforced concrete beams are widely used building members. The dimensions of flexural members are commonly selected so as to limit the deflections to acceptable limits. Though, in recent years, the tendency of studying deflections in reinforced concrete beams has resulted in the use of shallower sections. That is why, the question of deflection predicting for shallow reinforced concrete members has gained more importance [1-4] than that of the deep ones [5-16].

Both long-term and short-term deflections should be taken into design considerations. Due to using higher strength materials, dimensions of beams became less, which means, the question of deflection turn out to be the total design controlling aspect. Short-term deflection is defined as that happens instantaneously due to load application. The time is supposed to be insignificant, if the load is applied within some hours. Long-term deflection is that happens at some time period after the primary load application. A combination of creep and shrinkage, or concrete plastic-flow under sustained load, leads to the time-dependent phenomenon. Time-dependent or long-term deflection happens only when member is subjected to sustained loads.

It is not easy to calculate deflection of reinforced concrete beams with high accuracy. Reinforced concrete is an anisotropic, nonhomogeneous, inelastic, and nonlinear material. Consequently, deflection calculation classical methods will not lead to high accuracy results. The concrete elasticity modulus changes as a sustained load time function. This change is a many different variables function, such as, age of the concrete when loaded, temperature, humidity, load level, aggregate type, cement type etc… [1&2]. Thus, it is not easy to estimate accurately the concrete elasticity modulus, Ec. The change in the concrete elasticity modulus with time may rapidly increase it. Because the concrete tensile strength is not high, reinforced concrete beam cracks when subjected to service loads. The cracked section moment of inertia is very different from that of an uncracked one, because it relies on the crack length. Nevertheless, between the cracks, the section of concrete must be considered as uncracked. Finally, it should be said that because of cracking, the moment of inertia differs along the member span [3&4]. Based on the above, the current study suggests load-deflections expressions for differently loaded beams with different reinforcement ratios.

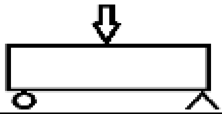
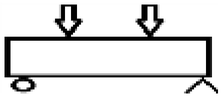
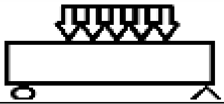
2. EXPERIMENTAL PROGRAM

In this paper, nine RC simply supported beams have been cast in the Structural Engineering Laboratory, College of Engineering at the University of Diyala. All beams have the same dimensions and reinforcement. They have an overall length of 1500 mm, a height of 250 mm and a width of 150 mm.

The properties of concrete mix used in the specimens are summarized in Table 1. These properties are concrete compressive strength f_c' [17 & 18], splitting tensile strength f_{ct} [19], modulus of rupture f_r [20], and modulus of elasticity $E_c = 4700\sqrt{f'c}$.

The reinforcement details, geometry and loading of the tested beams are shown in Figures 1-9.

Table 1 Concrete properties of the specimens

Group	Specimen Designation	f'_c (MPa)	f_t (MPa)	f_r (MPa)	E_c (MPa)	Type of Loading
A	1P-min 1P-ave 1P-max	24	2.5	4.337	23025	
B	2P-min 2P-ave 2P-max	32	3.2	5.5	26587	
C	Dist-min Dist-ave Dist-max	32	3.7	5	26587	

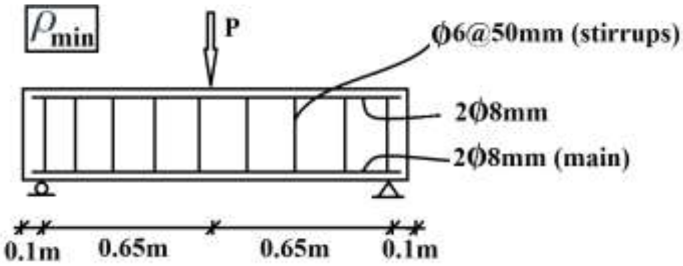


Figure 1 Reinforcement, geometry and loading of the beam 1P-min

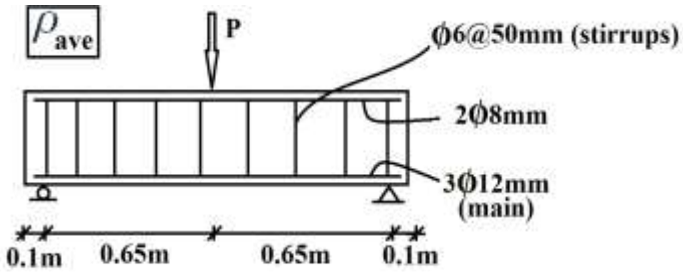


Figure 2 Reinforcement, geometry and loading of the beam 1P-ave

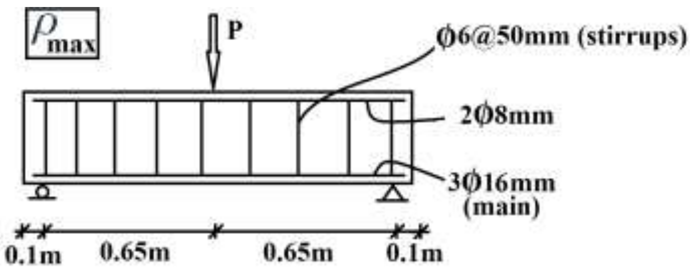


Figure 3 Reinforcement, geometry and loading of the beam 1P-max

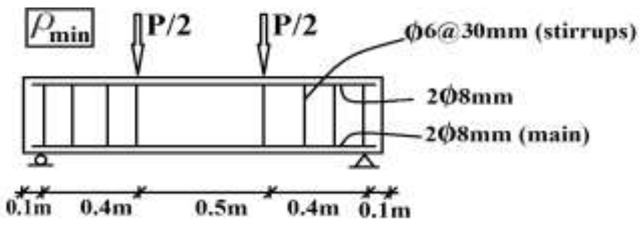


Figure 4 Reinforcement, geometry and loading of the beam 2P-min

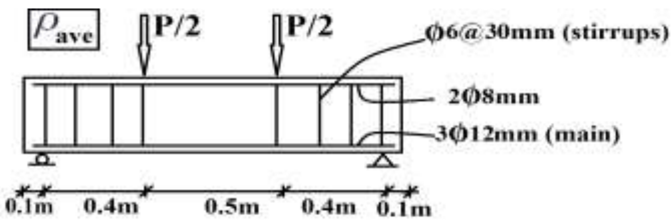


Figure 5 Reinforcement, geometry and loading of the beam 2P-ave

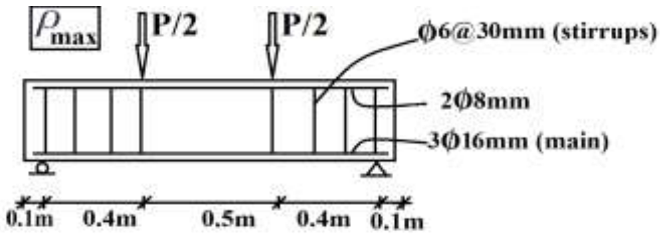


Figure 6 Reinforcement, geometry and loading of the beam 2P-max

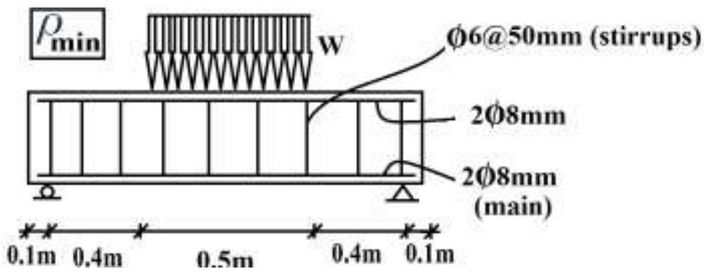


Figure 7 Reinforcement, geometry and loading of the beam Dist-min

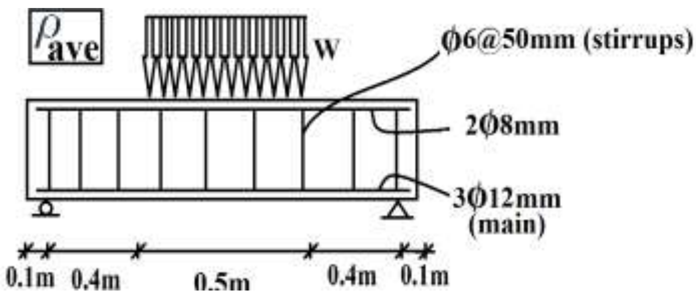


Figure 8 Reinforcement, geometry and loading of the beam Dist-ave

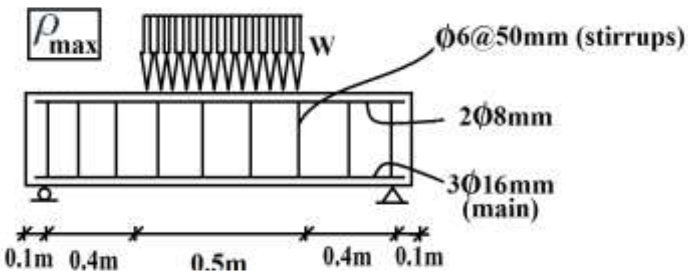


Figure 9 Reinforcement, geometry and loading of the beam Dist-max

3. TEST RESULTS AND DISCUSSIONS

Table 2 shows the test results in detail. Where P_f is the first flexural crack load, P_s is the first shear crack load, P_u is the ultimate load, Δ is the maximum mid-span defection, Δ_f is the mid-span defection at first flexural crack load and Δ_s is the mid-span defection at first shear crack load.

Table 2 Test results

Group	beams Desig-nation	f'_c (MPa)	P_f (kN)	P_s (kN)	Pu (kN)	Incr-ease in Pu %	Δ (mm)	Dec-rease in Δ %	$\frac{P_f}{P_u}$	$\frac{P_s}{P_u}$	$\frac{\Delta_f}{\Delta}$	$\frac{\Delta_s}{\Delta}$	Failure Type
A	1P-min	24	30	54	66	-	3	-	0.45	0.82	0.04	0.15	Flexure
	1P-ave		32	61	159	141	2.2	27	0.20	0.38	0.01	0.15	Flexure
	1P-max		51	72	251	280	2	33	0.20	0.29	0.07	0.10	Shear
B	2P-min	33	30	54	87	-	2.970	-	0.34	0.62	0.02	0.10	Flexure
	2P-ave		45	80	217	149	2.500	14	0.20	0.37	0.05	0.13	Flexure
	2P-max		60	40	312	258	1.440	50	0.19	0.13	0.12	0.10	Shear
C	Dist-min	33	35	65	87.5	-	3.026	-	0.4	0.74	0.05	0.19	Flexure
	Dist-ave		40	70	225	157	2.235	25	0.16	0.28	0.02	0.12	Flexure
	Dist-max		40	30	340	289	2.165	28	0.12	0.10	0.07	0.10	Shear

3.1. Cracks Propagation and Mode of Failure

3.1.1 Specimens of Group A

The first visible flexural cracks appeared in the mid-span zone (tension steel level) at load level about 45% and 20% of the ultimate capacity (at 4% and 1% of maximum mid-span deflection) for the beams 1P-min and 1P-ave, respectively. As the loads are increased, diagonal cracks appeared at about 82% and 39% of the ultimate capacity for the beams 1P-min and 1P-ave, respectively. As the loads are further increased, the flexural cracks extended towards the mid depth of the beam until reaching the load region as shown in Figures 10-11.

For the beam 1P-max, the first visible shear cracks appeared in the mid-span zone (tension steel level) at load level about 20% of the ultimate capacity (at 7% of maximum mid-span deflection). As the loads are increased, diagonal cracks appeared at about 29% of the ultimate capacity (at 10% of maximum mid-span deflection). As the loads are further increased, the cracks extended towards the mid depth of the beam until reaching the load region as shown in Figure 12.

For the beams 1P-min and 1P-ave, the failure occurred by splitting the beam into two parts, approximately along the mid-span line (flexural failure).

For the beam 1P-max, the failure occurred by splitting the beam into two parts near the support zone (shear failure).

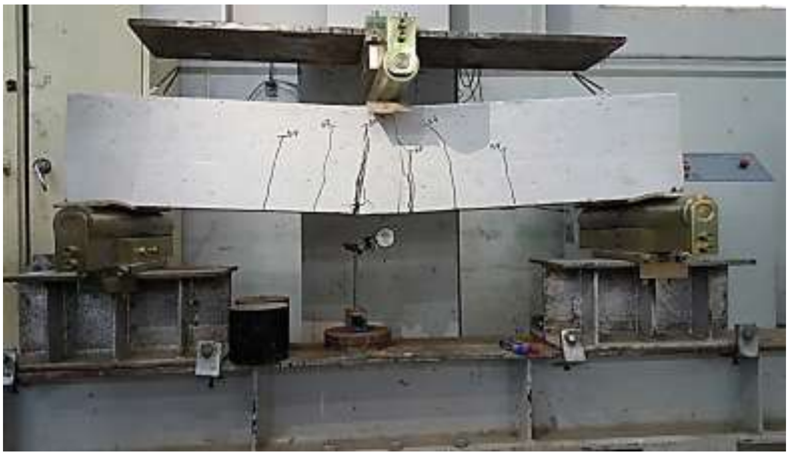


Figure 10 Cracks propagation for the beam 1P-min

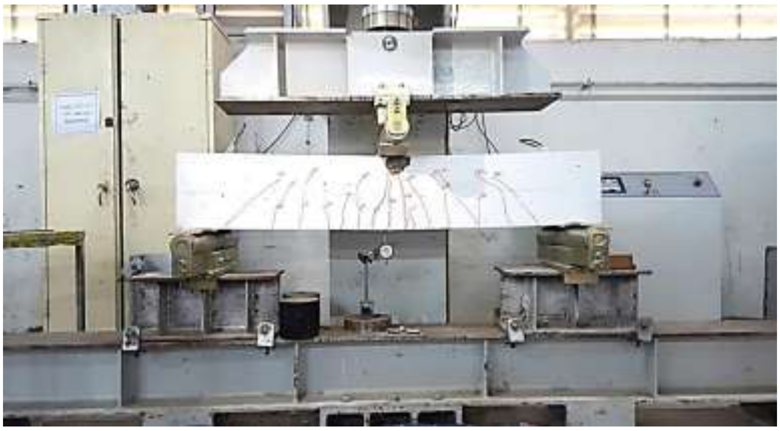


Figure 11 Cracks propagation for the beam 1P-ave

Figure 12 Cracks propagation for the beam 1P-max

3.1.2 Specimens of Group B

The first visible flexural cracks appeared in the mid-span zone (tension steel level) at load level about 34% and 20% of the ultimate capacity (at 2% and 5% of maximum mid-span deflection) for the beams 2P-min and 2P-ave, respectively. As the loads are increased, diagonal cracks appeared at about 62% and 37% of the ultimate capacity for the beams 2P-min and 2P-ave, respectively. As the loads are further increased, the flexural cracks extended towards the mid depth of the beam until reaching the load region as shown in Figures 13-14.

For the beam 2P-max, the first visible shear cracks appeared near the support (tension steel level) at load level about 13% of the ultimate capacity (at 10% of maximum mid-span deflection). As the loads are increased, flexural cracks appeared at about 19% of the ultimate capacity (at 12% of maximum mid-span deflection). As the loads are further increased, the cracks extended towards the mid depth of the beam until reaching the load region as shown in Figure 15.

For the beams 2P-min and 2P-ave, the failure occurred by splitting the beam into two parts, approximately along the mid-span line (flexural failure).

For the beam 2P-max, the failure occurred by splitting the beam into two parts near the support zone (shear failure).

Figure 13 Cracks propagation for the beam 2P-min

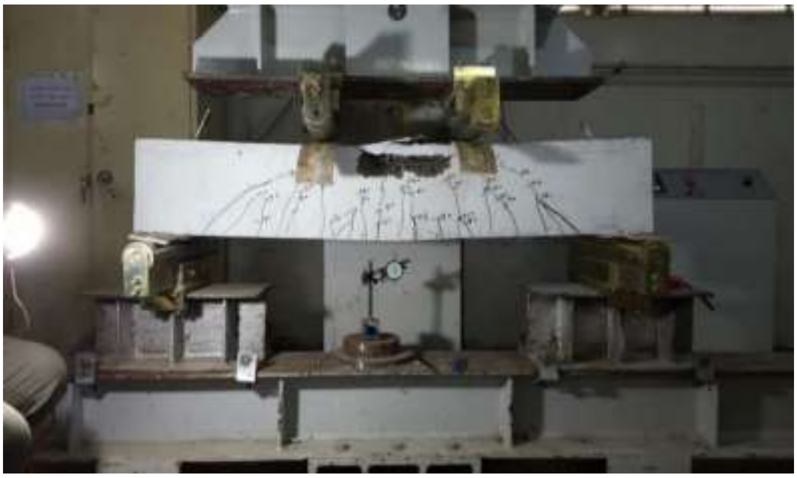


Figure 14 Cracks propagation for the beam 2P-ave

Figure 15 Cracks propagation for the beam 2P-max

3.1.3 Specimens of Group C

The first visible flexural cracks appeared in the mid-span zone (tension steel level) at load level about 40% and 18% of the ultimate capacity (at 5% and 2% of maximum mid-span deflection) for the beams Dist-min and Dist-ave, respectively. As the loads are increased, diagonal cracks appeared at about 74% and 31% of the ultimate capacity for the beams Dist-min and Dist-ave, respectively. As the loads are further increased, the flexural cracks extended towards the mid depth of the beam until reaching the load region as shown in Figures 16&17.

For the beam Dist-max, the first visible shear cracks appeared near the support (tension steel level) at load level about 10% of the ultimate capacity (at 7% of maximum mid-span deflection). As the loads are increased, flexural cracks appeared at about 12% of the ultimate capacity (at 10% of maximum mid-span deflection). As the loads are further increased, the cracks extended towards the mid depth of the beam until reaching the load region as shown in Figure 18.

For the beams Dist-min and Dist-ave, the failure occurred by splitting the beam into two parts, approximately along the mid-span line (flexural failure).

For the beam Dist-max, the failure occurred by splitting the beam into two parts near the support zone (shear failure).

Figure 16 Cracks propagation for the beam Dist-min

Figure 17 Cracks propagation for the beam Dist-ave

Figure 18 Cracks propagation for the beam Dist-max

3.2. Load Deflection Relationships

3.2.1 Specimens of Group A

It is found that ultimate capacity (P_u) increases and deflection (Δ) decreases about 141% and 27%, respectively, for the beam 1P-ave in comparison with the beam 1P-min because of using average reinforcement ratio instead of minimum one. For the beam 1P-max, in which the maximum steel reinforcement ratio is used, the increment in ultimate capacity (P_u) and decrement in deflection (Δ) become 280% and 33%, respectively in comparison with 1P-min. Figure 19 shows the load-deflection relationships for the specimens of Group A.

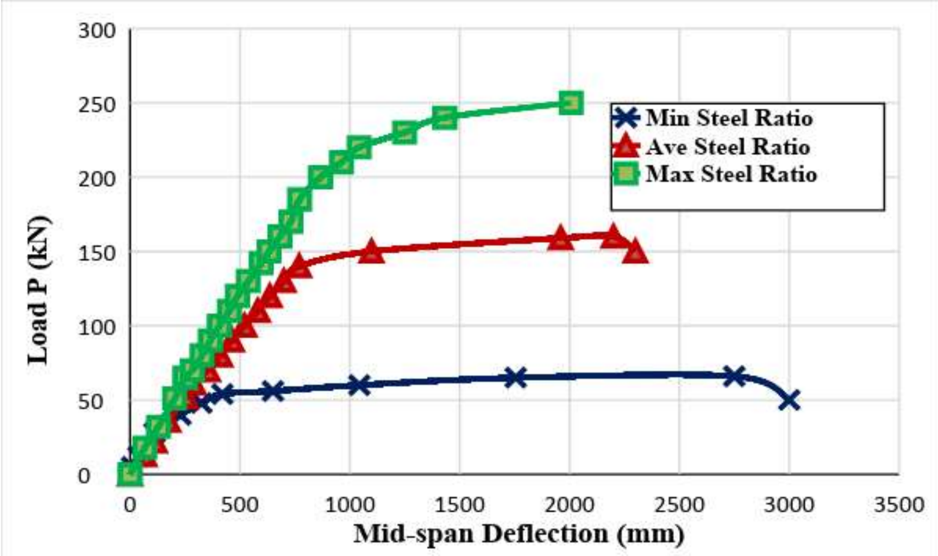


Figure 18 Load-midspan deflection for the specimens of Group A

3.2.2 Specimens of Group B

The ultimate capacity (P_u) increases and deflection (Δ) decreases by about 149% and 14%, respectively for the beam 2P-ave in comparison with the beam 2P-min because of using average reinforcement ratio instead of minimum one. For the beam 2P-max, in which the maximum steel reinforcement ratio is used, the increment in ultimate capacity (P_u) and decrement in deflection (Δ) become 258% and 50%, respectively in comparison with beam 1P-min. Figure 19 shows the load-deflection relationships for the specimens of Group B.

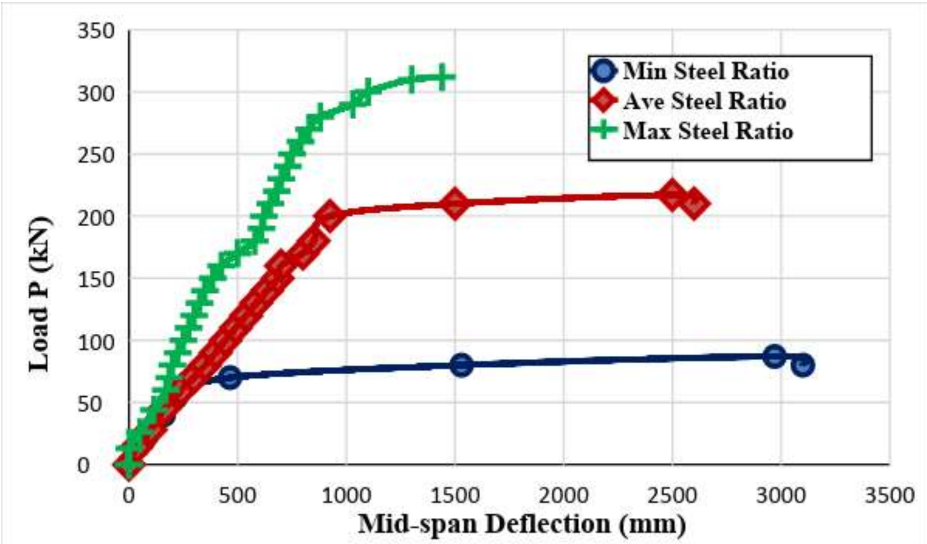


Figure 19 Load-midspan deflection for the Group B beams

3.2.3. Specimens of Group C

It is found that ultimate capacity (P_u) increases and deflection (Δ) decreases by about 157% and 25%, respectively for the beam Dist-ave in comparison with the beam Dist-min because of using average reinforcement ratio instead of minimum one. For the beam Dist-max, in which the maximum steel reinforcement ratio is used, the increment in ultimate capacity (P_u) and decrement in deflection (Δ) become 289% and 28%, respectively in comparison with beam Dist-min. Figure 20 shows the load-deflection relationships for the specimens of Group C.

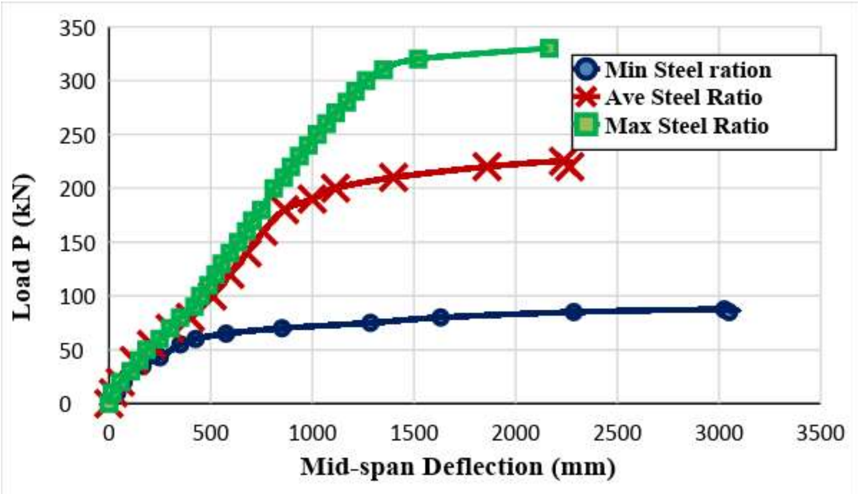


Figure 20 Load-midspan deflection for the specimens of Group C

3.3. First Crack Width

In the current experimental program, the first crack width besides their loads are detected, logged, drawn and discussed to investigate serviceability performance. The cracks expressed the manner that how the stresses affected the beams. More specifically, they presented data about how the loads went through the tested beams earlier and later the involvement of reinforcement. Figures 21-23 show the width of the first cracks that took place in the tested beams.

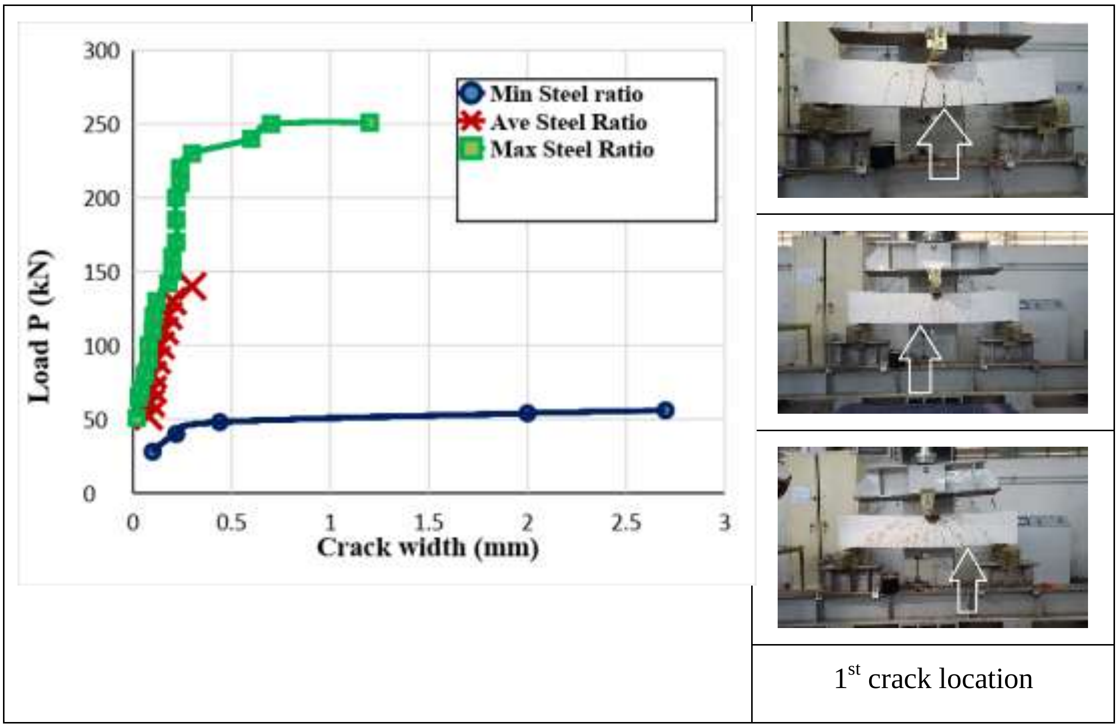


Figure 21 Load-first crack width for the specimens of Group A

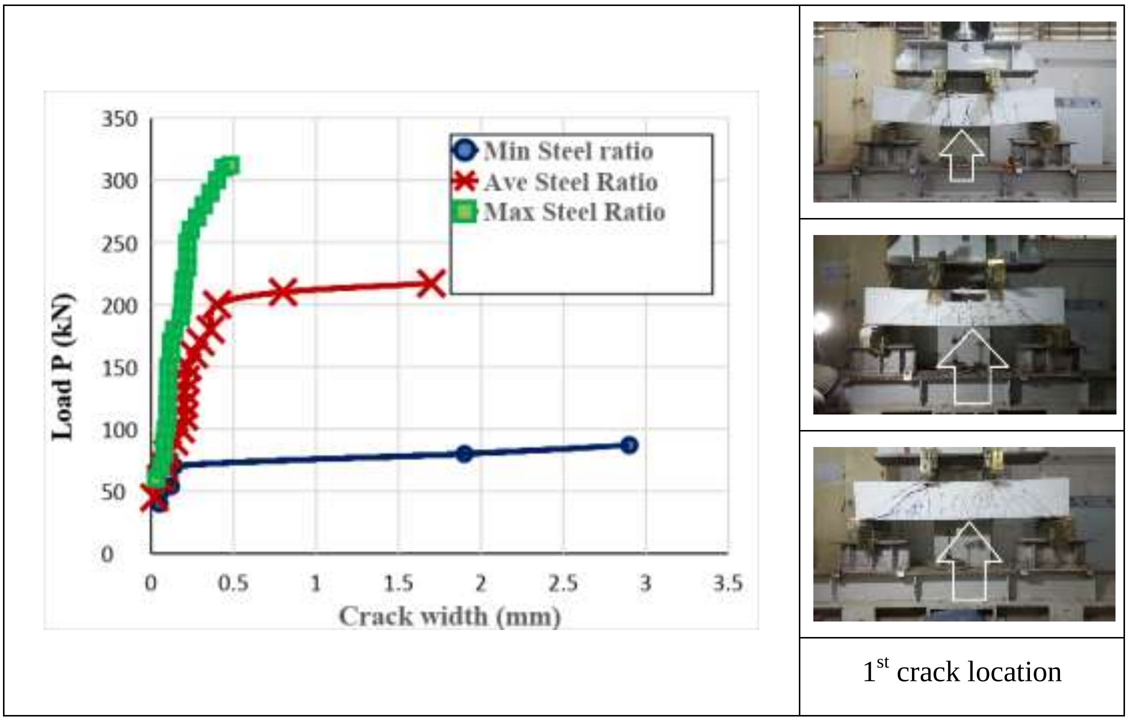


Figure 22 Load-first crack width for the specimens of Group B

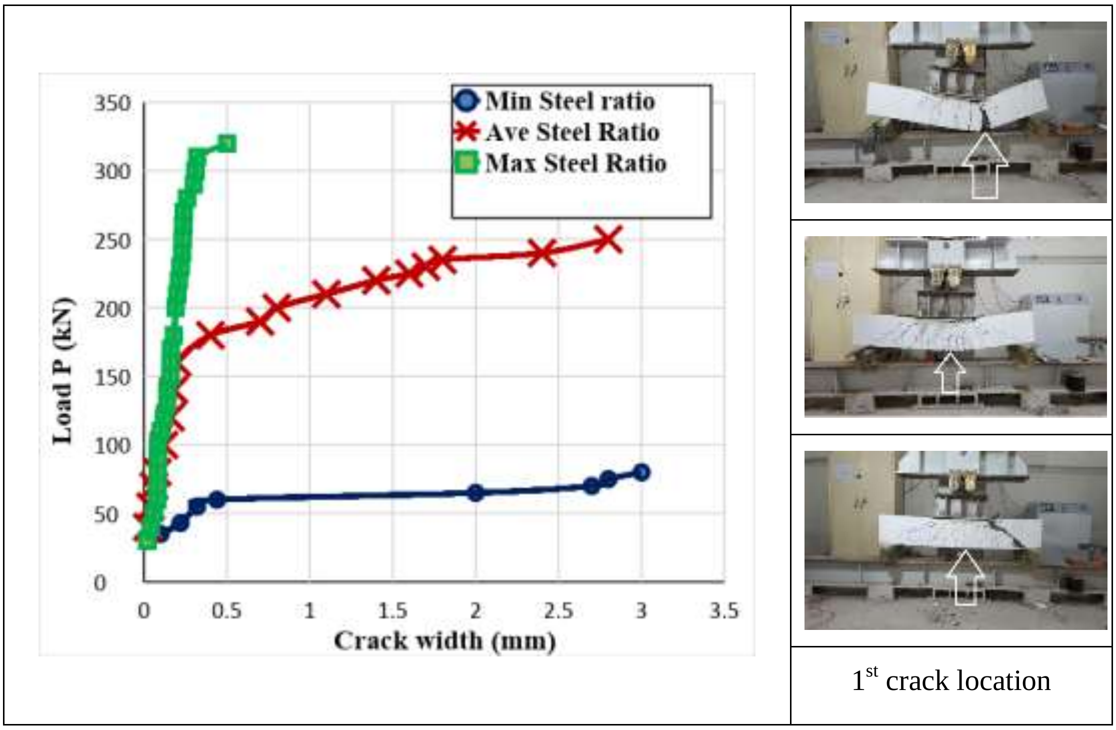


Figure 23 Load-first crack width for the specimens of Group C

4. Suggested Expressions for Load-Deflection

The following inferences are drawn based on the experimental investigations:

4.1. The specimens of Group A

In general, increasing the reinforcing steel ratio leads to increasing the ultimate capacity of the beam in addition to decreasing the maximum central deflection. So, increasing the reinforcing steel ratio from the minimum to the average of the minimum and the maximum leads to increase ultimate capacity by 141% and decrease maximum central deflection by 27%. Whereas increasing the reinforcing steel ratio from the minimum to the maximum leads to increase ultimate capacity by 280% and decrease maximum central deflection by 33%.

For the beams of a single concentrated force load, the relationships between load and deflection for the three reinforcing steel ratios are listed in Table 3.

Table 3 Relationships between load and deflection for Group A beams

Steel ratio	Relationship	R^2
Maximum	$y = -1E\text{-}16x^6 + 7E\text{-}13x^5 - 1E\text{-}09x^4 + 1E\text{-}06x^3 - 0.0005x^2 + 0.3181x - 0.9839$	0.9994
Average	$y = -4E\text{-}14x^5 + 3E\text{-}10x^4 - 5E\text{-}07x^3 + 0.0003x^2 + 0.1487x + 1.789$	0.9982
Minimum	$y = -2E\text{-}11x^4 + 1E\text{-}07x^3 - 0.0002x^2 + 0.1913x + 4.4714$	0.9844

Where y is the load and x are the mid-span deflection

4.2. The specimens of Group B

In general, increasing the reinforcing steel ratio leads to increase the ultimate capacity of the beam in addition to decrease the maximum central deflection. So, increasing the reinforcing steel ratio from the minimum to the average of the minimum and the maximum leads to increase ultimate capacity by about 150% and decrease maximum central deflection by about 14%. Whereas increasing the reinforcing steel ratio from the minimum to the maximum leads to increase ultimate capacity by 258% and to decrease maximum central deflection by 50%.

For the beams of 2-concentrated forces load, the relationships between load and deflection for the three reinforcing steel ratios are listed in Table 4.

Table 4 Relationships between load and deflection for Group B beams

Steel ratio	Relationship	R^2
Maximum	$y = 2E\text{-}13x^5 - 9E\text{-}10x^4 + 1E\text{-}06x^3 - 0.0007x^2 + 0.4923x + 2.4666$	0.9934
Average	$y = -2E\text{-}14x^5 + 2E\text{-}10x^4 - 4E\text{-}07x^3 + 0.0003x^2 + 0.1477x + 7.4306$	0.9954
Minimum	$y = -2E\text{-}11x^4 + 1E\text{-}07x^3 - 0.0003x^2 + 0.258x + 3.9293$	0.9893

Where y is the load and x is the mid-span deflection

4.3. The specimens of Group C

In general, increasing the reinforcing steel ratio leads to increase the ultimate capacity of the beam in addition to decrease the maximum central deflection. So, increasing the reinforcing steel ratio from the minimum to the average of the minimum and the maximum leads to increase ultimate capacity by 157% and decrease maximum central deflection by 25%. Whereas increasing the reinforcing steel ratio from the minimum to the maximum leads to increase ultimate capacity by 289% and to decrease maximum central deflection by 28%.

For the beams of a uniformly distributed load, the relationships between load and deflection for the three reinforcing steel ratios are listed in Table 5.

Table 5 Relationships between load and deflection for Group C beams

Steel ratio	Relationship	R^2
Maximum	$y = -7E\text{-}17x^6 + 5E\text{-}13x^5 - 1E\text{-}09x^4 + 1E\text{-}06x^3 - 0.0006x^2 + 0.3144x + 2.9676$	0.9997
Average	$y = 6E\text{-}15x^5 - 3E\text{-}12x^4 - 7E\text{-}08x^3 + 5E\text{-}05x^2 + 0.1916x + 6.9895$	0.9947
Minimum	$y = -1E\text{-}11x^4 + 8E\text{-}08x^3 - 0.0002x^2 + 0.1872x + 6.5014$	0.992

Where y is the load and x are the mid-span deflection

5. CONCLUSION

The purpose of the current experimental work is to study the behavior of reinforced beams under various steel ratios of ACI 318-14, which are the minimum (ρ_{min}), maximum (ρ_{max}) and the average of them ($\frac{\rho_{max}+\rho_{min}}{2}$). These different ratios are repeated three times under various loading types, which are single concentrated force, two-concentrated forces and a partial distributed load.

The following conclusions are drawn based on the experimental investigations:

Load-deflection expressions are presented here that can help design engineers.

For beams with single central concentrated force, the first cracks appear in the mid-span zone at load level about 45%, 20% and 20% of the ultimate for the beams with minimum, average and maximum steel ratio, respectively. It is found that increasing the reinforcing steel ratio from the minimum to the maximum leads to increase ultimate capacity by 280% and decrease maximum central deflection by 33%.

For beams with two concentrated forces, the first cracks appear in the mid-span zone at load level about 34%, 20% and 13% of the ultimate for the beams with minimum, average and maximum steel ratio, respectively. It is also found that increasing the reinforcing steel ratio from the minimum to the maximum leads to increase ultimate capacity by 258% and to decrease maximum central deflection by 50%.

For beams with uniformly distributed load, the first cracks appear in the mid-span zone at load level about 40%, 18% and 10% of the ultimate for the beams with minimum, average and maximum steel ratio, respectively. It is also found that increasing the reinforcing steel ratio from the minimum to the maximum leads to increase ultimate capacity by 289% and to decrease maximum central deflection by 28%.

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