



MULTIVARIABLE MATHEMATICAL MODELING AND SIMULATION OF DYNAMIC BEHAVIOR OF AN AIRCONDITIONING ROOM

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ABSTRACT

The aim of this study is to obtain and analyze the nonlinear multivariable mathematical model of the dynamic behavior of the system in order to save energy and apply modern control theories in air conditioning systems. For this, an experiment facility was established and the required mathematical model was prepared based on this installation. Control sizes considered in this model temperature, humidity and air velocity are variables. In addition to these, the mixing ratio formed by recirculation of air is also taken into account. Control elements include heaters, humidifiers, fans and mix air dampers. In this works; determining the internal and external variables of the system, defining the thermodynamic properties of humid air, determining the optimum continuous regime conditions, writing the system's energy, humidity and wall energy balance equations, establishing a multivariable mathematical model and simulating this model for the system. While establishing the mathematical model of the system, necessary assumptions were made to simplify the equations, the thermodynamic properties of the mix zone were determined and a solution was presented. In the nonlinear simulation results, iterative solutions have been made in a certain step range according to a suitable formulation and have been shown to exhibit a stable behavior. Mixing the air used in the system at a certain rate and returning it to the system by circulating does not disturb the stability of the system. The graphics of the studies are given and the results are interpreted. According to the mathematics model and simulation study established here, the results are very promising.

Key words: Air conditioning chamber, Optimal control theory, Multivariable systems, Mathematical modeling

Cite this Article: Yilmaz Akguney, Multivariable Mathematical Modeling and Simulation of Dynamic Behavior of an Airconditioning Room, *International Journal of Advanced Research in Engineering and Technology*, 11(6), 2020, pp. 502-517.

<http://www.iaeme.com/IJARET/issues.asp?JType=IJARET&VType=11&IType=6>

1. INTRODUCTION

In this study, the necessary mathematical model that can be used in the application of optimum control for air conditioning environmental space is established and the behavior of this model is examined. For this, I apply a suitable theory to a real-sized experiment room established by us [20] and interpret the results. In order to achieve this, the math model of a real multi-input and multi-output experiment room suitable for the linear model using the state variables approach was established in this study. Experimental results and simulation results of this model were obtained and interpreted. In this study, the mathematical model in A. Kaya's work [19, 21 and 22] was applied here and the studies were completed based on the simulation studies of E. Öznergiz [19].

The chamber of the air conditioning system is shown in Figure 2.1 and the cross-sectional view is shown in Figure 2.2 Basic approaches to control of such systems were introduced in the 1960s [1, 2, 3]. An analog simulation was developed for control. Simplified analytical techniques and analog simulation model are presented using frequency characteristic methods and clustering [4]. Temperature control has been studied to save energy [5, 6]. The enthalpy controller was produced [7], but this method could not hold the volume within the comfort limits [5, 6]. In addition, the dynamics of the humidifier were determined and its effect on heating costs was investigated [8]. Effects on comfort conditions have been investigated [9, 10, 11]. Comfort conditions are determined in a general relationship [12, 13]. Many optimization techniques have been proposed for the optimal solution of the closed loop control problem [14]. Models that give comfort conditions and use energy balance equations have been described [11]. A three-variable model has been developed that keeps volume variables constant within the comfort zone [15]. A model created without taking into account the air circulation and the stability of the model established in the past is also shown [16]. The energy use function and optimum control problem regarding comfort conditions have been developed and solved on the basis of an example [17]. Some experimental results are given for energy saving [18]. A mathematical model is created for the control variables of the air-conditioning room and simulation results are given [19]. Considering all the studies and deficiencies carried out so far, a air-conditioning room has been constructed in appropriate dimensions and experimental conditions and the results have been interpreted [20]. Control principles of effective energy use are presented in a air conditioning room [21]. Modeling of an experiment room with minimum energy is presented [22]. A room was investigated by establishing a heat transfer correlation for energy modeling [23]. The performance of an experiment room operating with minimum energy use has been investigated [24]. Two optimization techniques have been proposed for optimum control of energy use in comfort conditions [25]. In this study, the automatic control equipment of the test system modeled and simulated has been introduced [26]. Later, many people worked on the experimental system [20], which was established by me in 1989 and completed by 1994 for my doctoral thesis studies.

2. MATERIAL AND METHOD

The features of the test chamber (Figures 1 and 2) are detailed in reference [20, 21, 22, 23, 24, 25 and 26]. Temperature, humidity and air velocity measurement elements, heater, humidifier, fan and damper motor control and computer-to-room information transfer are given in references [24, 28 and 29]. In the system, the outlet air is given back to the entrance to meet the fresh air need of the room. In the construction of the control room, 0.018 m thick fiberboard was used. Inlet, outlet and return air ducts are made of 0.65 mm galvanized sheet. The lower and upper surfaces of the room and the ducts were insulated, while other surfaces

only provided insulation that is proportional to the heat transfer coefficient of the chipboard. The surfaces of the chipboard material are painted with oil paint in order not to absorb moisture. The air velocity is uniformly distributed with the grilles placed on the ceiling and floor. Some features and dimensions of the room given in Figures 2.1 and 2.2 are as follows. Air duct section is 0.4x0.5m. The wall thickness is 0.018 m, the density of the wall material is 500 kg/m³, the specific temperature of the wall material is 2.5 kJ/kg⁰C, the heat transmission coefficient of the wall material is 2.5 kJ/hm⁰C, the horizontal cross section of the room is 2 m², the volume of the room is 3.6 m³, the insulated surface area of the room is 10.8 m², respectively, the inner and outer surface film coefficients are 50 and 30 kJ/m²h⁰C. Since there is no standard value for the properties of the wall material, approximate results have been obtained. The heat transfer coefficients on the inner and outer surfaces are dependent on the velocity, density, etc. of the air experience values are used. However since these studies are not healthy studies, reference is not given here.



Figure 1 The view of the test chamber

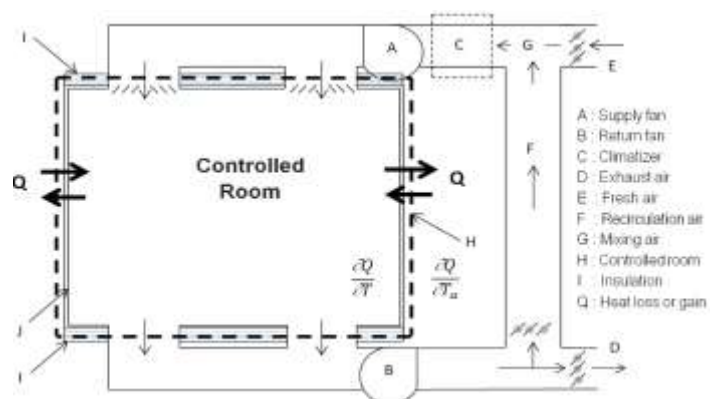


Figure 2 The cross-sectional view of the test chamber

3. MULTIVARIABLE MATHEMATICAL MODEL INSTALLATION OF THE SYSTEM'S DYNAMIC BEHAVIOR

In the controlled system, the relevant element is air and the thermodynamic properties of the air are used in the equations used. Some assumptions are made in the system. Pressure variation is accepted in fixed and external conditions. There is no air leak in the system. Heat transfer is only through the side walls. Radiation heat transfer is not available and convection heat transfer coefficients are constant. Air distribution within the control volume is uniform. There is no element that exchanges moisture and heat.

3.1. Thermodynamic Properties of Moist Air

Air is always present mixed with some moisture (water vapor). The most important subject of air conditioning technique is the thermodynamic properties of humid air. In other words, air, which is a mixture of many gases, is considered only as a mixture of dry air and water vapor and the condition of this mixture is examined under constant pressure and at different temperatures.

3.1.1. Relative Humidity

Table 1 Relative humidity formulation

Explanation	Equation	Equ. Nu.
The ratio of the partial pressure of the water vapor in the air the partial pressure of the water vapor in saturated air at the same conditions (at the same temperature and total pressure) is called relative humidity. This ratio is equal to the ratio of the moisture mass in the air to the moisture mass in saturated air at the same temperature, equation (1). If the ideal gas laws are written for water vapor and saturated steam, respectively, equations (2) and (3) are obtained and if they are proportioned, equation (4) is obtained. $\bar{T} = T + 273.15$ is taken.	$\phi = \frac{P_w}{P_s}$	(1)
	$P_w V = m_w R_w \bar{T}$	(2)
	$P_s V = m_s R_w \bar{T}$	(3)
	$\phi = \frac{P_w}{P_s} = \frac{m_w}{m_s}$	(4)

3.1.2. Specific Moisture

Table 2 Specific moisture formulation

Explanation	Equation	Equ. Nu.
The water vapor weight in dry air per unit weight is called specific humidity. Its value is usually given as kg water vapor/kg dry air. It can also be defined as the ratio of the mass of water vapor in the mixture to the dry air mass. Ideal gas laws for water vapor and dry air are written as in (6) and (7), respectively. Since the temperatures of the gases are the same and the volume they will cover according to the Dalton-Gibbs law will be regarded as equal, (6) and (7) if the ratio of masses is drawn, the equation is obtained (8). The expression (9) is obtained by substituting the gas constant values of dry air and water vapor in (8). According to the Dalton-Gibbs law of (10), the pressure of the gas mixture is written in (10) as (11) using equations (4), (9) and (10). The saturation pressures of the water vapor at the given temperatures are found in the thermodynamic tables. If it is necessary to express the saturation pressure as a function of temperature, the correlation is used in the SI unit system (12) between 0 and 50 °C. Here, $\alpha=652.747$ (Pa) and $\beta=0.06115$ (1/°C) are constants. The variation of the saturation pressure according to the temperature can be considered to be exponential in this way. After defining the logarithm of the exponential function, after defining it as a linear change, the coefficients α and β are determined by the numerical solution [19, 20]. The variation of the saturation pressure, as shown in the tables, α and β is calculated and found with a great approach. Thus, equation (11) can be written like (13) [15, 17]. This expression can be written as relative humidity as in (14).	$w = \frac{m_w}{m_a}$	(5)
	$P_w V = m_w R_w \bar{T}$	(6)
	$P_a V = m_a R_a \bar{T}$	(7)
	$w = \frac{m_w}{m_a} = \frac{R_a P_w}{R_w P_a}$	(8)
	$w = 0.622 \frac{P_w}{P_a}$	(9)
	$P = P_w + P_a$	(10)
	$w = 0.622 \frac{\phi P_s}{P - \phi P_s}$	(11)
	$P_s = \alpha e^{\beta T}$	(12)
	$w = 0.622 \frac{\phi \alpha e^{\beta T}}{P - \phi \alpha e^{\beta T}}$	(13)
	$\phi = \frac{P w}{(0.622 + w) \alpha e^{\beta T}}$	(14)

3.1.3. Density of Humid Air

Table 3 Formulation of moist air density

Explanation	Equation	Equ. Nu.
By using $P v = R \bar{T}$ ideal gases law, (15) equation can be written. In addition, the correlation (11) can be arranged as (16) and $\bar{w} = w + 0.622$. If (17) equality is put in place (15), the result of equation (18) is reached. Maximum and minimum specific humidity within the comfort zone limits in the system is approximately 0.003 to 0.012 kg water vapor/kg dry air $w \geq 0.622$. If the average specific humidity of 0.008 kg of water/kg of dry air is taken by neglecting the effect of specific humidity on the density, (19) equality can be written.	$\frac{1}{\rho} = R_a \frac{\bar{T}}{P - P_w}$	(15)
	$P_w = \phi P_s = P \frac{v}{\bar{w}}$	(16)
	$P - P_w = P \left(1 - \frac{w}{\bar{w}}\right) = \frac{63.0241}{\bar{w}}$	(17)
	$\rho = \frac{219.596}{\bar{v} \bar{T}}$	(18)
	$\rho = \frac{348.289}{\bar{T}}$	(19)

3.1.4. Enthalpy of Moist Air

Table 4 Formulation of moist air enthalpy

Explanation	Equation	Equ. Nu.
The enthalpy of moist air, which is saturated, is the sum of enthalpy of dry air and enthalpy of water vapor (20). If we show the enthalpy of the mixture containing 1 kg of dry air and w kg of water vapor with h, we can write (21). If 0 °C temperature is chosen as the reference point, the enthalpy of dry air (22) can be calculated from the equation. Since the specific temperature of the air changes very slowly with the temperature, it can be considered constant for temperatures not exceeding 100 °C. In this case, $c_{pa} = 1.0035$ kJ/kgK can be taken in company with (22). For more sensitive calculations, the change of specific heat with temperature can be considered. The triple point temperature of the water is 0.01 °C and it is very close to 0 °C. By neglecting the difference, we can assume that the enthalpy of water at 0 °C is equal to zero. The saturation pressure of water at 0 °C is 0.6108 kPa. It is an ideal gas of water vapor in moist air. Therefore, the enthalpy of water vapor is considered to be a function of temperature only. Instead of the enthalpy of water vapor at the P_w partial pressure and T temperature in the mixture, enthalpy of the water vapor at T temperature of 0.6108 kPa and T temperature can be taken [20]. Therefore, the equation (23) can be written here. An approximate solution of this expression is to find c_{pw} and h_g values using the numerical solution method. If the expression of equality (23) is substituted in (21), the expression (24) can be written for the enthalpy of humid air (mixture) containing 1 kg of dry air and w kg of water vapor. If expression (31) is to be written for outdoor air, (25) correlation is reached [24].	$H = m_a h_a + m_w h_w$	(20)
	$h = h_a + w h_w$	(21)
	$h_a = c_{pa} T$	(22)
	$h_w = h_g + c_{pw} T$	(23)
	$h = c_{pa} T + w(h_g + c_{pw} T)$	(24)
	$h_a = c_{pa} T_a + w_a(h_g + c_{pw} T_a)$	(25)

3.1.5. Enthalpy of Water

Table 5 Formulation of enthalpy of water

Explanation	Equation	Equ. Nu.
There is compressed water in the humidifier. The enthalpy (26) of the water here is written as equation. Since h_w is the specific enthalpy of compressed water, the enthalpy of water (27) can be written as an equation. Assuming that the specific temperature of the cw water does not depend on the temperature, the average value of $c_w = 4.1878$ kJ/kgK is used [20].	$H_w = m_w h_w$	(26)
	$H_w = m_w c_w T_w$	(27)

3.2. Determination of Optimum Continuous Regime Conditions

3.2.1. Features of the Air Entering the Control Room

In air conditioning, it is necessary to calculate the specific humidity, enthalpy and temperature of the mixture obtained by adiabatic mixing of air masses with different humidity rates under different conditions. The ratio of the mass of the return air to the mass of the air entering the control room is indicated by r , and the properties of the air entering the control room are determined by the formulation below.

Table 6 Formulation of the characteristics of air entering the control room

Explanation	Equation	Equ. Nu.
The flow of air returning to the control room is as in equality (28). At the same time, the mass flow of the return air is as in Equality (29). The fresh air flow is as in equation (30). The mass flow of fresh air is as in equality (31). Since the flow rate of the mixing air is f , the mass flow of the air leaving the control room is $\rho_e f$. In this case, the mass flow of the air leaving the room in the continuity equation of dry air is equal to the sum of the mass flow of the return air as in equation (32). In the continuity equation in the water vapor, the mass flow of water vapor leaving the room is equal to the sum of the mass flow of water vapor of the return air and the mass flow of water vapor of fresh air (equation 33). Equality (33) can be written as in (34). Equality (32) can be written as (35). It is obtained by using expressions (19) and (35) (36). The enthalpy of the air leaving the volume is the sum of the enthalpy of the return air and the energy of the fresh air (37). The expression (37) is written in terms of the enthalpy of the mixed air (38). Also, equation (32) is substituted here and correlation (39) is obtained.	$f_r = r f$	(28)
	$\rho f_r = \rho r f$	(29)
	$f_e = (1 - r) f$	(30)
	$\rho_a f_e = \rho_a (1 - r) f$	(31)
	$\rho_e = \rho r + \rho_a (1 - r)$	(32)
	$w_e \rho_e = \rho r w + \rho_a (1 - r) w_a$	(33)
	$w_e = \frac{\rho r w + \rho_a (1 - r) w_a}{\rho_e}$	(34)
	$w_e = \frac{\rho r w + \rho_a (1 - r) w_a}{\rho r + \rho_a (1 - r)}$	(35)
	$w_e = \frac{348.289 r w + \rho_a (1 - r) w_a \bar{T}}{348.289 r + \rho_a (1 - r) \bar{T}}$	(36)
	$\rho_e h_e = \rho r h + \rho_a (1 - r) h_a$	(37)
	$h_e = \frac{\rho r h + \rho_a (1 - r) h_a}{\rho_e}$	(38)
	$h_e = \frac{\rho r h + \rho_a (1 - r) h_a}{\rho r + \rho_a (1 - r)}$	(39)

3.2.2. Determination of the Amount of Moisture to be Supplied

Table 7 Formulation of determining the amount of moisture to be given

Explanation	Equation	Equ. Nu.
After the effects of the control values in the air entering the control room, the moisture balance value for the system becomes as in the equations (40) and (41) for the control room. If equations (35), (40) and (41) are regulated, (42) and (43) are obtained. If the expression (42) is arranged with the help of the equality (43), the expression (44) is obtained. If (44) is rearranged, (45) is obtained. By using equation (45) in correlation (19), (46) is obtained. Equality (45) can be written as (47) with the help of the equality obtained as a result of these processes. The expression (13) is written instead of w_a . In this way, the amount of moisture required in the continuous regime is expressed in terms of other variables and equation (48) is obtained. $\lambda_a = \phi_a \alpha e^{\beta T_a}$, [15, 17 and 18].	$w = w_e + \frac{m}{M}$	(40)
	$M = A v \rho$	(41)
	$w = \frac{r \rho w + (1-r)\rho_a w_a}{r \rho + (1-r)\rho_a} + \frac{m \bar{T}(1-r)\rho_a}{A v 348.285 r \rho + (1-r)\rho_a}$	(42)
	$w = 1 - \frac{r \rho}{r \rho (1-r)\rho_a}$	(43)
	$w \left(1 - \frac{r \rho}{r \rho + (1-r)\rho_a} \right) = \left(1 - \frac{r \rho}{r \rho (1-r)\rho_a} \right) w_a + \frac{m}{A v \rho}$	(44)
	$w = w_a + \frac{r \rho + (1-r)\rho_a}{(1-r)\rho_a A v \rho} m$	(45)
	$\frac{r \rho + (1-r)\rho_a}{(1-r)\rho_a A v \rho} = \frac{r(\bar{T}_a - \bar{T}) + \bar{T}}{(1-r)\bar{T}}$	(46)
	$w = w_a + \frac{[\bar{T} + r(T_a - T)]m}{348.289 A v (1-r)}$	(47)
	$m = \left(w - \frac{0.622 \phi_a \alpha e^{\beta T_a}}{P - \phi_a \alpha e^{\beta T_a}} \right) \frac{348.289 A v (1-r)}{\bar{T} + r(T_a - T)}$	(48)

3.2.3. Determination of Amount of Heat to be Supplied

Table 8 Formulation of the amount of heat to be delivered

Explanation	Equation	Equ. Nu.
After the effects of the control variables in the air entering the room, the energy balance value for the system becomes like the equation (49) for the control room. The expression of heat loss in the steady state in the system should be substituted as in equation (50). The expression (51) is obtained by using equations (39), (49) and (50). If necessary intermediate operations are performed (52) are obtained. Here the equality (24) equality is written and the expression (13) is used. The equation (53) is obtained for the amount of heat to be given.	$h = h_e + \frac{q}{M} - \frac{Q}{M}$	(49)
	$Q = K S (T - T_a)$	(50)
	$h = \frac{r \rho h + (1-r)\rho_a h_a}{r \rho + (1-r)\rho_a} + \frac{q - K S (T - T_a)}{M}$	(51)
	$h = \frac{\bar{T} + r(T_a - T)}{348.289 A v (1-r)} [q - K S (T - T_a)] + h_a$	(52)
	$q = \frac{348.289 A v (1-r)}{\bar{T} + r(T_a - T)} [c_{pa}(T - T_a) + w \{ c_{pw} T + h_g - \frac{0.622 \lambda_a}{P - \lambda_a} (c_{pw} T_a + h_g) \}] + K S (T - T_a)$	(53)

3.2.4. Determination of Mixture Temperature

Table 9 Formulation of determining the mixture temperature

Explanation	Equation	Equ. Nu.
Based on the equation (24), the enthalpy expression of the mixing air becomes the same as in the equation (54). This expression is written as T_e (55). It is obtained by using equations (34), (39) and (55) and by making intermediate operations (56). Using the expressions (19), (56), (24) and (25) and after the necessary intermediate operations, the expression (57) is concluded.	$h_e = c_{pa} T_e + w_e (h_g + c_{pw} T_e)$	(54)
	$T_e = \frac{h_e - w_e h_g}{(c_{pa} + w_e c_{pw})}$	(55)
	$T_e = \frac{[r \rho h + (1-r) \rho_a h_a] - [r \rho w + (1-r) \rho_a w_a] h_g}{[r \rho + (1-r) \rho_a] c_{pa} + [r \rho_w + (1-r) \rho_a] c_{pw}}$	(56)
	$T_e = \frac{348.289 r [c_{pa} + w c_{pw}] T + (1-r) \bar{T} \rho_a [c_{pa} + w_a c_{pw}] T_a}{348.289 r [c_{pa} + w c_{pw}] + (1-r) \bar{T} \rho_a [c_{pa} + w_a c_{pw}]}$	(57)

3.3. System Equations

3.3.1. Balance Equation of Humidity

It is assumed that it is not a moisture absorbing agent in the control room. In this case, the sum of the moisture amount in the air entering the system and the water vapor given is equal to the humidity change in the control room and the amount of moisture in the exiting air. For the system shown in Figures 1 and 2, mathematically moisture balance can be written as follows.

Table 10 System's moisture balance formulation

Explanation	Equation	Equ. Nu.
$\left[\begin{array}{c} \text{Moisture} \\ \text{(water vapor)} \\ \text{difference} \\ V \rho \frac{dw}{dt} \end{array} \right] = \left[\begin{array}{c} \text{Flow of} \\ \text{humidifier} \\ \text{(evaporating} \\ \text{water)} \\ \dot{m} \end{array} \right] + \left[\begin{array}{c} \text{Amount of} \\ \text{moisture in} \\ \text{mixed inlet} \\ \text{air} \\ w_e \rho_e f \end{array} \right] - \left[\begin{array}{c} \text{Amount of} \\ \text{moisture in the} \\ \text{exiting air} \\ w \rho d \end{array} \right]$		(58)
$d = Av$. If we divide this expression by $v\rho$ and put it in $1/V$ common parenthesis, equation (59) is obtained.	$\frac{dw}{dt} = \frac{1}{V} \left\{ \frac{\dot{m}}{\rho} + \frac{f w_e \rho_e}{\rho} - w A v \right\}$	(59)
Equations (13) and (15) have the expression (60) if used in the equation (59).	$\frac{dw}{dt} = \frac{1}{V} \left\{ \dot{m} (K_1 \bar{T} \bar{w}) + f \frac{(0.622 P_{we}) (P - P_{we}) / \bar{T}_e}{(P - P_{we}) (P - P_w) / \bar{T}} - A v w \right\}$	(60)
By making the necessary abbreviations in Equality (61), moisture balance equation for this system can be written as in (62).	$\frac{dw}{dt} = \frac{1}{V} \left\{ K_1 \dot{m} \bar{T} \bar{w} + f (0.622 \lambda_e) \frac{\bar{T}}{\bar{T}_e} \frac{1}{P - \left(P \frac{w}{\bar{w}} \right)} - A v w \right\}$	(61)
	$\frac{dw}{dt} = \frac{1}{V} \left\{ K_1 \dot{m} \bar{T} \bar{w} + \frac{f \bar{T} \bar{w}}{P \bar{T}_e} \lambda_e - A v w \right\}$	(62)

3.3.2. Balance Equation of Energy

Table 11 Formulation of the energy balance equation

Explanation and Equation	Equ. Nu
<p>The change in the energy level of the air in the control room can be written in this way.</p> $\left[\begin{array}{c} \text{Difference} \\ \text{in air energy} \\ \text{(enthalpy)} \\ V \rho \frac{dh}{dt} \end{array} \right] = \left[\begin{array}{c} \text{Amount of} \\ \text{heat energy} \\ \text{supplied} \\ q \end{array} \right] + \left[\begin{array}{c} \text{Energy} \\ \text{amount of} \\ \text{mixing air} \\ \text{(inlet air)} \\ h_e \rho_e f \end{array} \right] + \left[\begin{array}{c} \text{Humidifier} \\ \text{water} \\ \text{energy} \\ m h_w \end{array} \right] - \left[\begin{array}{c} \text{Energy} \\ \text{in the} \\ \text{exiting} \\ \text{air} \\ h \rho d \end{array} \right] - \left[\begin{array}{c} \text{Amount of} \\ \text{heat transfer} \\ \text{through the} \\ \text{wall} \\ Q \end{array} \right]$	(63)
<p>(63) is written in the form and in this statement necessary arrangements are made as follows.</p> $\rho V (c_{pa} + c_{pw} w) \frac{dT}{dt} = q + \rho_e f h_e + m h_w - Q - \rho d h - \rho V (h_g + c_{pw} T) \frac{dw}{dt}$	(64)
<p>The expression (62) is substituted here.</p> $\frac{dT}{dt} = \frac{1}{V(c_{pa} + c_{pw} w)} \left\{ \frac{q}{\rho} + f h_e \frac{\rho_e}{\rho} + \frac{m h_w}{\rho} - A v h - \frac{Q}{\rho} - V (h_g + c_{pw} T) \frac{dw}{dt} \right\}$	(65)
$\frac{dT}{dt} = \frac{1}{V(c_{pa} + c_{pw} w)} \left\{ q K_1 \bar{T} \bar{w} + f [c_{pa} T_e + w_e (h_g + c_{pw} T_e)] \bar{T} \bar{w} / \bar{T}_e \bar{w}_e + m h_w K_1 \bar{T} \bar{w} - A v h - Q K_1 \bar{T} \bar{w} - V (h_g + c_{pw} T) \frac{dw}{dt} \right\}$	(66)
<p>For the control room under study, the energy balance equation can be written as here.</p> $\frac{dT}{dt} = \frac{K_1 \bar{T} \bar{w}}{V (c_{pa} + c_{pw} w)} \left[q + \frac{f E}{K_1} + m h_w - Q \right] - \frac{A v [c_{pa} T + w (h_g + c_{pw} T)]}{(c_{pa} + c_{pw} w)} - \frac{(h_g + c_{pw} T)}{V (c_{pa} + c_{pw} w)} \left[K_1 m \bar{T} \bar{w} + \frac{f \bar{T} \bar{w}}{P \bar{T}_e} \lambda_e - A v w \right]$	(67)

3.3.3. Balance Equation of Mass (Velocity Relation)

Kontrol odasına giren ve çıkan hava için süreklilik denklemi eşitlik (68)'deki gibi yazılabilir. Gerekli ara işlemler yapıldıktan sonra, sistem için hava hızı bağıntısı eşitlik (73) şeklini alır.

Table 12 Formulation of balance equation of mass (velocity relation)

Equation	Equ. Nu.	Equation	Equ. Nu.
$f \rho_e = A v \rho$	(68)	$v = \frac{f \bar{T} P - \phi_e \alpha e^{\beta T_e}}{A \bar{T}_e P - P w / \bar{w}}$	(71)
$v = \frac{1 \rho_e}{A \rho} f$	(69)	$v = \frac{f (P - \lambda_e) \bar{T} \bar{w}}{A (0.622 P) \bar{T}_e}$	(72)
$v = \frac{f (P - P_{we}) / \bar{T}_e}{A (P - P_w) / \bar{T}}$	(70)	$v = \frac{f (P - \lambda_e) \bar{T} \bar{w}}{A K_2 \bar{T}_e}$	(73)

3.3.4. Heat Transfer Relation

Referans [20 ve 23] de, burada incelenen deney tesisatının duvarları için gerekli ısı transfer modellenmesi detaylı bir şekilde yapılarak verilmiştir. Sürekli rejim durumunda duvarlar için ısı kaybı ifadesi olarak eşitlik (75) kullanılabilir.

Table 13 Formulation of heat transfer relation

Equation	Equ. Nu.
$\frac{d\dot{Q}}{dt} = \frac{1}{R_1} \frac{dT}{dt} + \frac{1}{C_d R_1 R_2}$ $[T - T_a] - \frac{\dot{Q}}{C_d} \left[\frac{R_1 + R_2}{R_1 R_2} \right]$	(74)
$Q_{ss} = \frac{S}{R_1 + R_2} (T - T_a)$ $= U S (T - T_a)$	(75)

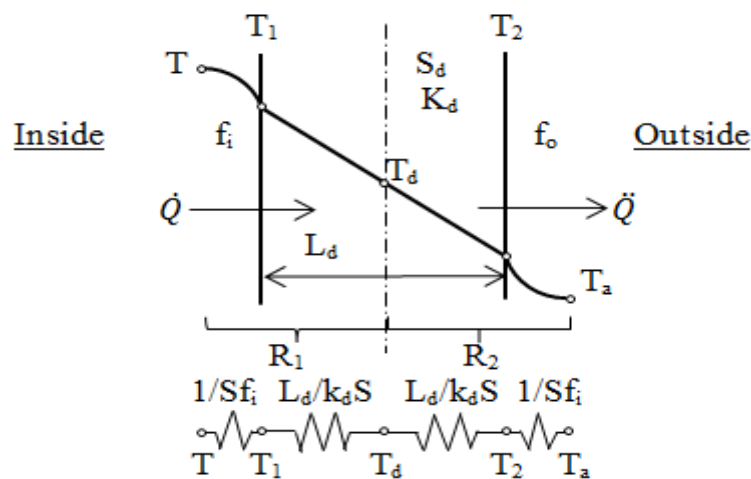


Figure 3 Heat transfer section of the wall

4. SYSTEM SIMULATION

The Runge-Kutta IV method was used based on the studies in the reference [19]. This method solves iteratively according to the formulation below and according to the h step range. Sensitivity is around 10^{-5} . By using the previous value in each iteration, it is possible to find the value of the variable at any time until the end of the calculation time. The error increases for the same step size as the calculation time increases. For this, the number of steps should not be chosen too small. The maximum and minimum values in these expressions are the time constant and natural frequencies of these different systems, considering the multivariable systems. For systems in the form of a time constant, the h step range can be between 90-110 and $100/\zeta$ for vibrating systems. In this case, the most appropriate solution should be sought within the above limits. In the Runge-Kutta IV program used for this, the number of variables can be changed as desired and it is expected to give correct results. Due to the number of variables, the number of differential equations can be changed as desired. In this system, the number of differential equations are energy balance, moisture balance and wall energy balance. These are the equations (62), (67) and (74) that come out of these equilibrium equations and give time-dependent changes of temperature, humidity and heat loss. Since the velocity variable, which is taken as the state variable, is not a differential change, this change is recalculated in every iteration in terms of other sizes. Simulation results are taken for the following values. The selection of these values results from the maximum values in the system. Heater power is maximum 14000 kJ/h and humidifier power is maximum 4 kg/h. In this case, the simulation is done for sizes below these values.

Table 14 Runge-kutta IV formulation [19]

The first conditions are the Runge-kutta IV formulation of x_0, t_0 and differential equation.		
$\frac{dx}{dt} = f(t, x)$ including,	$K_1 = h f(t_0, x_0)$	$K_2 = h f(t_0 + \frac{1}{2}h, x_0 + \frac{1}{2}K_1)$
$K_3 = h f(t_0 + \frac{1}{2}h, x_0 + \frac{1}{2}K_2)$	$K_4 = h f(t_0 + h, x_0 + K_3)$	$t = t_0 + h$ variable x value instantly
Step size and calculated range (Δt) are selected. The accuracy of the result of the simulation depends on them. For appropriate results, these values are selected as follows.		
For time constant type vibrations,		$\Delta t > 4 \tau_{max}, \quad h \leq \frac{\tau_{min}}{10}$
For systems of vibration element type,		$\Delta t > \frac{4}{\zeta w_{0min}}, \quad h \leq \frac{2\pi}{50 w_{0max}}$

5. RESULTS AND DISCUSSION

5.1. Results

The necessary mathematical model has been established and evaluated to apply modern control theories to a multivariate air conditioning system. It was carried out as follows. Using the state variables approach, the mathematical model of a physical system, which was established under the conditions of the multi-input and multi-output experiment room suitable for the linear model, was established. The stability of the system was demonstrated by solving the behavior of this installed model with Runge-Kutta IV. The mathematical infrastructure required for a real problem to be used digitally with modern computer control techniques is presented here. In the simulation study, the established nonlinear mathematical model was examined and the behavior of the system was interpreted by solving the nonlinear equation sets of the behavior of this nonlinear system. In the program used in the simulation study, variables appropriate to the way the program will run are defined. A number between 0 and 1 is determined for the mixing ratio. Since the calculation steps are reduced and calculated in itself, there are no rounding errors. The following values were used in simulation studies. $\dot{Q} = 14000$ kJ/h, $\dot{m} = 0$ (heating), $\dot{Q} = 14000$ kJ/h, $\dot{m} = 4$ kg/h (heating + humidification) and $\dot{Q} = 0, \dot{m} = 4$ kg/h (humidification).

The graphical variation of some of the results obtained is given below. All simulation studies have shown that the system exhibits similarity and decisive behavior in terms of behavior. System; circulation, the amount of air deemed suitable for back circulation to the room as mixing air, did not have a negative effect on its stable operation. In Table Appendix-2, important values used in comparing the simulation results of the nonlinear equation sets and experiment results in various climatic conditions and mixture ratios are given.

5.2. Discussion

Here, the nonlinear mathematical model established and the nonlinear equation sets of the behavior of this nonlinear system have been solved and the system behavior has been examined. Analytical solutions of nonlinear equation sets are difficult. These solutions need to be reexamined for different initial conditions and parameters. Iterative operations can be done with the help of computer. Numerical solution methods are an alternative to this situation. Another alternative solution is the establishment of the linear model. However, linearization only provides correct solutions around the initial conditions and in a narrow range. In case of getting away from the initial conditions, the errors increase. Consequently, in every initial condition change and if the state variables diverge from the initial conditions, the linearization should be done again in the new conditions. When these solutions are examined for general equation sets, analytical solution is possible for several variable algebraic and linear differential equation sets. However, it is not possible for all multivariate sets of

equations. Nonlinear systems are only possible for several variable algebraic and ordinary differential equations, but still difficult. In numerical solution methods, the solution of the equation sets is easily used with approximate solutions by using computers. Therefore, the iterative studies carried out make these methods efficient using a computer. Using linear models, operations are performed with matrices as the number of variables increases. Manual operation cannot be done here. The desired result is obtained only if a computer is used. This method is especially used in modern control methods.

It is seen here that modern control can be dealt with in a real problem and control application of the model can be done in digital environment. The mathematical model and simulation results used here are similar to the time constant system. The system exhibits a stable behavior in terms of behavior. In the system, the use of some of the return air by returning inside did not cause an unstable operation. In the system presented here, it is seen that mathematical modeling, control and simulation results are extremely suitable.

6. SYMBOLS

A	Cross-sectional area of chamber (m ²)	P	Atmospheric pressure (kPa)
c _{pa}	Air specific heat (kJ/kgK)	P _a	Partial pressure of dry air (kPa)
c _{pw}	Specific heat of water vapor (kJ/kgK)	P _s	Partial pressure of vapor in saturated air (kPa)
c _w	Specific heat of wall (kJ/kg K)	P _w	Partial pressure of water vapor in humid air (kPa)
c _w	Specific heat of water (kJ / kgK)	q	Heat control variable (kJ/h)
d	Average airflow at outlet (m ³ /h)	Q	Heat loss (kJ/h)
f	Total air flow (m ³ /h)	Q _w	Absorbed heat on the wall, (kJ/h).
f _e	Fresh air flow (m ³ / h)	Q _i	Heat transfer from the inner wall through conduction, (kJ/h)
f _{e,r}	Return air flow (m ³ /h)	Q _t	Heat transfer by convection on the inner wall surface, (kJ/h).
f _i	The film coefficient of the inner surface of the wall, (kJ/m ² h ⁰ C)	r	Return air mixture ratio (%)
f ₀	Outer surface film coefficient of wall, (kJ/m ² h ⁰ C).	m _w	Mass of moisture
H	Enthalpy of humid air (kJ)	R _a	Gas constant for air (kJ/kgK)
H _w	Enthalpy of moisture water (kJ)	R	Gaz sabiti (kJ/kgK)
h _a	Enthalpy of dry air (kJ / kg)	R _w	Gas constant of water vapor (kJ / kgK)
h	Room air enthalpy content (kJ/kg)	R ₁ , R ₂	Thermal resistances, (m ² h ⁰ C/kJ)
h _e	Enthalpy of the mixed air (kJ / kg)	R _w	Vapor gas constant
h _g	Specific enthalpy of water vapor (kJ/kgK)	r	Air mixing ratio (%)
h _w	Enthalpy of the water (kJ/kg)	S	Room area, Wall heat loss surface area (m ²)
K	Total heat transfer coefficient of the wall, (kJ/m ² h ⁰ C)	T	Time (h)
k _w	Heat conduction coefficient of the wall, (kJ/m ² h ⁰ C).	T	Room temperature (°C)
L _w	Total wall thickness, (m).	T̄	T + 273.15 (°C)
M	Mass flow rate, (kg/h)	T ₁	Internal surface temperature of the wall, (°C)
m	Humidity control variable (kg/h)	T ₂	Wall, outside surface temperature, (°C).
m _a	Mass of dry air (kg)	T _e	Mixed air temperature (°C)
m _s	Water vapor mass in saturated air (kg)	T _a	Outside or room temperature (°C)
m _w	Mass of moisture (kg)	T _w	Nemlendirici suyu sıcaklığı (°C)
P	Atmospheric pressure (kPa)	T _w	Water temperature (°C)
P _a	Partial pressure of dry air (kPa)	v	Air velocity in room (m/s)
P _s	Partial pressure of vapor in	v _a	Ambient air velocity, (m/s)
		v _r	Relative velocity of the air and the human body (m/s)
		V	Voluma of chamber (m ³)
		w	Moisture content (kg moisture/kg of dry air)
		w _a	Outdoor specific humidity (kg water / kg dry air)
		w _e	Specific humidity of the mixed air

P_w	saturated air (kPa) Partial pressure of water vapor in humid air (kPa)	ϕ	(kg water / kg dry air) Relative humidity (%)
q	Heat control variable (kJ/h)	ϕ_a	External air (ambient air) relative humidity, (%)
Q	Heat loss (kJ/h)	α	Constant, 652.747 (P_a)
\dot{Q}_w	Absorbed heat on the wall, (kJ/h).	β	Constant, 0.06115 ($1/^\circ\text{C}$)
\dot{Q}_i	Heat transfer from the inner wall through conduction, (kJ/h)	λ	$\phi \alpha e^{\beta T}$
\dot{Q}	Heat transfer by convection on the inner wall surface, (kJ/h).	ρ	Density of air (kg/m^3)
r	Return air mixture ratio (%)	ρ_a	Dış hava yoğunluğu (kg/m^3)
m_w	Mass of moisture	ρ_e	Density of entering air (kg/m^3)
T_w	The temperature in the middle of the wall, ($^\circ\text{C}$)	ρ_d	The average density of the wall material, (kg/m^3)
T_m	Room average radiation Temp. ($^\circ\text{C}$)	ρ_w	Density of wall material (kg/m^3)
		υ	Özgül hacim (m^3/kg)

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APPENDIX

Table Appendix-1 Change of water vapor saturation pressure for $P_{wh} = \alpha e^{\beta T}$ as a function of temperature.

Constants: By taking $\alpha = 652.747$ (Pa), $\beta = 0.06115$ (1/°C)											
T (°C)	P _w (Pa)	P _{wh} (Pa)	Error (%)	T (°C)	P _w (Pa)	P _{wh} (Pa)	Error (%)	T (°C)	P _w (Pa)	P _{wh} (Pa)	Error (%)
0.0	610.758	652.747	6.875	17.0	1936.323	1846.159	4.656	34.0	5318.146	5221.476	1.817
1.0	656.457	693.913	-5.075	18.0	2012.238	1962.588	2.467	35.0	5622.152	5550.774	1.269
2.0	705.392	757.696	-4.577	19.0	2195.709	2086.361	5.207	36.0	5939.888	5900.839	0.657
3.0	757.368	784.197	-3.804	20.0	2336.925	2217.940	5.091	37.0	6274.295	6272.982	0.021
4.0	812.873	833.654	-2.556	21.0	2485.912	2357.817	5.118	38.0	6624.392	6668.594	-0.667
5.0	871.811	886.229	-1.654	22.0	2641.912	2506.515	5.140	39.0	6991.161	7089.156	-1.402
6.0	934.574	842.12	-0.807	23.0	2807.644	2664.591	5.368	40.0	7374.601	7536.241	-2.192
7.0	1001.259	1001.536	-0.027	24.0	2982.202	2832.636	5.015	41.0	7776.673	8011.522	-3.020
8.0	1072.063	1064.699	0.687	25.0	3166.567	3011.279	4.904	42.0	8198.678	8516.777	-3.884
9.0	1147.280	1131.845	1.345	26.0	3359.758	3201.189	4.719	43.0	8638.678	9053.896	-4.807
10.0	1277.106	1203.226	1.946	27.0	3563.737	3403.075	4.508	44.0	9099.591	9624.890	-5.773
11.0	1311.738	1279.109	2.487	28.0	3778.502	3617.693	4.256	45.0	9582.078	10331.894	-6.782
12.0	1401.468	1359.777	2.974	29.0	4004.055	3845.848	3.951	46.0	10085.160	10887.179	-7.853
13.0	1496.593	1445.533	3.412	30.0	4241.376	4088.389	3.607	47.0	10611.780	11563.159	-8.965
14.0	1597.405	1536.697	3.800	31.0	4491.446	4346.229	3.233	48.0	11161.930	12292.402	-10.127
15.0	1704.004	1633.610	4.131	32.0	4753.283	4620.328	2.797	49.0	11735.620	1306.636	-11.350
16.0	1816.878	1736.636	4.416	33.0	5028.850	4911.714	2.329	50.0	12334.800	13891.759	-12.612

Table Appendix-2 Some values used in comparison of simulation results and experiment results (Test and simulation conditions).

Experiment Number	T (°C)	T _a (°C)	φ (%)	φ _a (%)	D _a (°)	Y _U (V)	R (%)	f _r (m ³ /h)	T _w (°C)
HEATING v = 0.1 m/s f = 720 m ³ /h q = 14000 kJ/h m = 0									
1	20	19	60	60	0	10	100	720	-
2	22.7	19.5	40.2	42	18	8	90	648	-
3	22.2	20	44	45	36	6	73	525	-
4	21.9	20	43.1	45	54	4	45	324	-
5	22.2	20	42	45	72	2	15	108	-
6	24.4	20.5	27.4	30	90	0	0	0	-
HEATING + HUMIDIFYING v = 0.1 m/s f = 720 m ³ /h q = 14000 kJ/h m = 4 kg/h									
7	19.5	18.5	56	56	0	10	100	720	21
8	20	18.5	60.9	62	18	8	90	648	22.5
9	20.1	19	60.3	62	36	6	73	525	23
10	20.2	19	60.8	62	54	4	45	324	23.5
11	21.4	19.5	55	58	72	2	15	108	22
12	21.6	20	55.5	58	90	0	0	0	23
HUMIDIFYING v = 0.1 m/s f = 720 m ³ /h q = 0 m = 4 kg/h									
13	22.5	20.5	71.2	71.2	0	10	100	720	21
14	21.57	20.5	59.2	62	18	8	90	648	20.5
15	21.3	20.5	62	64	36	6	73	525	20
16	20.8	20.5	67.2	70	54	4	45	324	19
17	20.45	20	66.3	70	72	2	15	108	18.5
18	20.1	20	68.3	70	90	0	0	0	18
HEATING v = 0.2 m/s f = 1440 m ³ /h q = 14000 kJ/h m = 0									
19	20.8	20.5	40.2	40.2	0	10	100	1440	-
20	23.5	20.5	40.2	18.8	18	8	92.5	1332	-
21	23.4	21	44	30.4	36	6	80	1152	-
22	22.6	21	43.1	31	54	4	55	792	-
23	23	21.5	42	24.5	72	2	20	288	-
24	24.3	22	27.4	20	90	0	0	0	-
HEATING+HUMIDIFYING v = 0.2 m/s f = 1440 m ³ /h q = 14000 kJ/h m = 4 kg/h									
25	20.3	19.5	42	42	0	10	100	1440	20
26	20.8	20	49.46	49	18	8	92.5	1332	23
27	21.9	20.5	44	44.4	36	6	80	1152	23
28	23	20.5	33	33.4	54	4	55	792	23
29	22.8	21	38	38.2	72	2	20	288	22
30	22.8	21	45	45	90	0	0	0	22
HUMIDIFYING v = 0.2 m/s f = 1440 m ³ /h q = 0 m = 0.4 kg/h									
31	21.8	21.5	50	50	0	10	100	1440	20.5
32	21	21.5	50	51	18	8	92.5	1332	20
33	22.4	22.2	36.4	36.4	36	6	80	1152	19
34	22.64	22.3	40.9	41	54	4	55	792	18
35	20.9	20.9	42.3	42.3	72	2	20	288	17
36	21.3	21.3	44.36	44.2	90	0	0	0	17

Table Appendix-3 Examples of comparison of experiment and simulation results with graphics

