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OPTIMAL SERVER TIME MANAGEMENT IN A SINGLE SERVER QUEUE

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ABSTRACT

This paper considers an optimal server time management of a single server queue with T-policy in which the server is under vacation until the realization of a random variable T. When T is realized, the server takes all waiting customers as a batch for service. After the batch service completion, the server shifts to a single service if there is any waiting customer and become idle when the system becomes empty. Such types of models deal with a very important class of real life congestion situations encountered in day-to-day as well as industrial scenario. The problem is to find a policy for selecting the service rate which maximizes the expected net profit per cycle in an M/M/1/K queue. Steady-state solutions and various performance measures are derived and obtained the optimal service rate numerically.

Key words: Finite capacity, Markovian Queue, Optimization, Single or bulk service, T- policy

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1. INTRODUCTION

In this paper, we investigate the optimal server time management of a single server M/M/1/K queue with T-policy in which the server is under vacation until the realization of a random variable T. The server starts service on the realization of T and provides batch service to customers waiting in the system if there is any. After completing the batch service, the server turns in to single service to the customers who joined in the queue during the batch service time. Here T is acting as a threshold for the server to start service. The vacation time T of the server can be used efficiently by the managers by assigning different tasks. Also, if the manager wants to monitor continuously the queue for an arrival, the T-policy vacation can be utilized as a post processing time after clearing the jobs in the system. The successive monitor may result in high management cost. To reduce the management cost, the managers would prefer a T - policy. We can see similar situations in production and manufacturing systems, quality control problems etc. Hence the queue management is a smart, efficient way to balance costs, minimize loss, and avoid overinvesting in equipment, products, or labor hours we do not actually need and all these maintaining a high level of customer satisfaction.

Queuing models under various thresholds policies consider the most common concern of controlling influxes and in reducing down the total cost. The various applications of *thresholds models* can be made in day-to-day as well as industrial scenarios. For example, in a production *systems*, wherein an item proceeds through various work station considered as queuing system. The arriving items made at an assembly line where a worker has inactivity between successive jobs. To use the time effectively, production managers can assign secondary tasks to the employee. Some other situations like the communication and computer networks, these server vacations corresponding to a power saving mode, where the server is turned off in order to save energy. In many waiting line systems, mechanical/electronic devices provide the role of a server, such as computer, ATM, traffic light etc. which is subject to accidental waiting of customers. This may be solved by the server vacations randomly and due to batch service criteria. Last two decades have witnessed a tremendous growth in the applications of bulk service queues to many congestion situations. The main reason for this is such models are encountered in many real life systems such as transportation systems, computer systems, telecommunication, airline scheduling, industrial processes such as production/ industrial systems etc.

The T-policy for queuing systems have been introduced by the pioneering work of Heyman (1977), in which the server is turned off at the end of a busy period and scans the queue after T time units. Later, many studies on the variants of T-policy models have been done (See, Alfa and Frigui (1996), Ke (2005, 2006, 2008), Ke and Chu (2008, 2009), Tadj (2003), Wang and Ke (2002), Wang et.al (2009), Bart Feyaerts (2014). Kim and Moon (2006) provide a randomized T-policy to control the server in which, when the system is empty, the server can be switched off with probability p and take a vacation or left with probability $(1-p)$ and continue to serve the arriving customer. Also, Zang et.al. (2011) established the convexity of the cost function in T based on the service cycle for the M/G/1 queue with T-policy and Xuelu Zhang et.al. (2015) studied a T-threshold service policy in an M/M/1 queue and Khalid Alnowibet and Tadj (2019) analyzed two phase queue with vacations and breakdowns under T-policy.

The inclusion of vacation into bulk queuing models makes the model more feasible in practical situations. Very vast literature exist for such models (for details one can refer Li and Zhu (1996), Chaudhary (2002) and many others). As far as our knowledge goes, no work has reported in the literature for a single/bulk service queue with various batch size services under the random T policy. Providing bulk service to the customers in the queue, the waiting time of the customers in the queue reduces but this increases the holding cost of customers in the system. Also, if we activate the server each and every time arrival an occur leads to heavy running cost of the system. Hence, both extremes are not desirable and it is natural to look for a tradeoff between the two. Hence, we have introduced the T policy to activate the server.

This paper is organized as follows. Section 1 is an introduction section and section2 gives the mathematical modeling and analysis. In section3, we derived system characteristics and section 4 provides cost analysis and numerical illustrations.

2. MATHEMATICAL MODELING AND ANALYSIS

In this paper, we have considered a single arrival finite capacity queuing system under single/bulk service. Arrivals occur according to a Poisson process of rate λ and the server is under vacation under the realization of a random variable T, which is exponentially distributed with parameter α . On realization of T, the server starts service and provides group service to the customers waiting in the system. The maximum number of waiting customers in the system is K-1. If the random variable T is not realized until the (K-1) [the] arrival, then together with the K^{th} arrival, the server automatically starts batch service of size K. The number of customers under the batch service can be $2, \dots, K$. After completing the batch service, server turns into single service mode and provide a single service to the customers who joined during the batch service time. Once the server is in the active mode, it continues service until the system becomes empty and then goes for the vacation. The server returns to the active mode from the vacation after the realization of T and starts service if there are at least two customers present for service. Otherwise, the server goes for another vacation. That is, here we use a multiple vacation policy to the server. Also assumed that the new arrivals to the system is lost when the queue is full and the service times of units follow independent exponentially distributed random variables with service rates μ_1 and μ_2 respectively for single and batch service irrespective of the number of units in the batch and inter arrival times and service times are independent of each other.

Let $N_1(t)$ and $N_2(t)$ be the number of customers in the system and the number of customers in service at time t respectively. Also let $S(t)$ be the state of the server at time t and define

$$S(t) = \begin{cases} 0, \text{ if the server is idle at time t} \\ 1, \text{ if the server is under single service at time t} \\ 2, \text{ if the server is under group service at time t} \end{cases}$$

and let $Z(t) = (N_1(t), N_2(t), S(t)), t \geq 0$. Then, $\{Z(t), t \geq 0\}$ is a continuous time Markov chain over the finite state space E, where $E = \{(i,0,0) \mid 0 \leq i \leq K-1\} \cup \{(i,1,1) \mid 1 \leq i \leq K\} \cup \{(i,j,2) \mid 2 \leq j \leq i, 2 \leq i \leq K\}$.

Let us define $P_{i,j,r}(t) = P\{((N_1(t), N_2(t), S(t)) = (i,j,r)/(N_1(0), N_2(0), S(0)) = (0,0,0)\}$, $(i,j,r) \in E$.

2.1. Steady State Solution

Here the Markov chain $\{Z(t), t \geq 0\}$ is finite and irreducible, the limit distribution exists and let it be

$q_{i,j,r} = \lim_{t\to\infty} P_{i,j,r}(t), (i,j,r) \in E$. The steady state balance equations of the system are

$$0 = -\lambda\, q_{0,0,0} + \mu_1\, q_{1,1,1} + \mu_2 \sum_{i=2}^{K} q_{i,i,2} \quad (1)$$

$$0 = -(\lambda + \alpha)\, q_{i,0,0} + \lambda\, q_{i-1,0,0} \quad i = 1, 2, 3, \dots, K-1 \quad (2)$$

$$0 = -(\lambda + \mu_1) q_{i,1,1} + \lambda q_{i-1,1,1} + \mu_1\, q_{i+1,1,1} + \mu_2 \sum_{j=2}^{K-i} q_{i+j,j,2}, \; i = 2, 3, \dots, K-2 \quad (3)$$

$$0 = -(\lambda + \mu_1) q_{K-1,1,1} + \lambda q_{K-2,1,1} + \mu_1\, q_{K,1,1} \quad (4)$$

$$0 = -\mu_1 q_{K,1,1} + \lambda q_{K-1,1,1} \quad (5)$$

$$0 = -(\lambda + \mu_1)\, q_{1,1,1} + \mu_1\, q_{2,1,1} + \mu_2 \sum_{j=2}^{K-1} q_{j+1,j,2} + \alpha q_{1,0,0} \quad (6)$$

$$0 = -(\lambda + \mu_2) q_{i,i,2} + \alpha q_{i,i,0}, \text{i} = 2,3, \dots, \text{K}-1 \quad (7)$$

$$0 = -\mu_2 q_{K,K,2} + \lambda q_{K-1,0,0} \quad (8)$$

$$0 = -(\lambda + \mu_2) q_{i,j,2} + \lambda q_{i-1,j,2}, \text{i} = 3, 4, \dots, \text{K}; \; \text{j} = 2, 3, \dots, \text{i}-1 \quad (9)$$

Recursively solving the equations (1-9), we get the joint steady state distribution of the number of customers in the system, the number of customers in service and state of the server.

Equations (1) and (4) gives $q_{i,0,0} = (\frac{\lambda}{\lambda+\alpha})^i q_{0,0,0}$, $i = 1, 2, 3, \dots, K-1$ and (6) gives

$q_{i,i,2} = \left(\frac{\alpha}{\lambda+\mu_2}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^i q_{0,0,0},, \quad i = 2,3,\ldots,K-1$ and from (7)

$$q_{K,K,2} = \left(\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1} q_{0,0,0}.$$

Now from equation (8),

$$q_{i,j,2} = (\frac{\lambda}{\lambda+\mu_2})^{i-j}\left(\frac{\lambda}{\lambda+\alpha}\right)^j \frac{\alpha}{\lambda+\mu_2} q_{0,0,0}, \mathrm{i} = 3,4,\ldots,\mathrm{K}-1,\mathrm{K}; \ \ \mathrm{j} = 2,3,\ldots,\mathrm{i}-1$$

Equation (1) gives $q_{1,1,1}= \frac{\lambda}{\mu_1}\{1-\frac{\mu_2}{\lambda+\mu_2}\frac{\lambda}{\lambda+\alpha}\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\right]\}q_{0,0,0}$and equation (5) gives

$$q_{2,1,1} = \frac{\lambda}{\mu_1}\{\left(1+\frac{\lambda}{\mu_1}\right)\left[1-\frac{\mu_2}{\lambda+\mu_2}\frac{\lambda}{\lambda+\alpha}\left(1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\right)\right]-\frac{\mu_2}{\lambda+\alpha}\left(\frac{\lambda}{\lambda+\mu_2}\right)^2\left(1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-2}\right) - \frac{\alpha}{\lambda+\alpha}\}q_{0,0,0}$$

Now we can rewrite equation (2) as

$$q_{i+1,1,1}-\left(1+\frac{\lambda}{\mu_1}\right)q_{i,1,1}+\frac{\lambda}{\mu_1}q_{i-1,1,1} = \frac{-\mu_2}{\mu_1}\frac{\lambda}{\lambda+\alpha}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i+1}\left(1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-i-1}\right)q_{0,0,0},$$

for $\quad i = 2,3,\ldots,K-2$ (10)

where $\sum_{j=2}^{K-i} q_{i+j,j,2}$ gives the terms in the right hand side except the first term.

The homogeneous part of the equation is $q_{i+1,1,1}-\left(1+\frac{\lambda}{\mu_1}\right)q_{i,1,1}+\frac{\lambda}{\mu_1}q_{i-1,1,1} = 0$. Using the forward shift operator $Eq_{i,1,1} = q_{i+1,1,1}$, we get $E^2-\left(1+\frac{\lambda}{\mu_1}\right)E+\frac{\lambda}{\mu_1} = 0$, gives the roots of the equation are $\frac{\lambda}{\mu_1}$ or 1.

Hence the general solution of the equation (10) is

$q_{i,1,1} = A(\frac{\lambda}{\mu_1})^i + B - \frac{\frac{-\mu_2}{\mu_1}\frac{\lambda}{\lambda+\alpha}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i+1}\left(1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-i-1}\right)q_{0,0,0}}{r(\frac{\lambda}{\lambda+\mu_2})}, i = 3,\ldots,K-1$, where

$r\left(\frac{\lambda}{\lambda+\mu_2}\right) = \frac{\lambda}{\mu_1}\frac{\mu_2}{\lambda+\mu_2}\frac{\lambda+\mu_2-\mu_1}{\lambda+\mu_2}$. Since $\sum_{i=1}^{K} q_{i,1,1} < 1$, $B = 0$. Therefore

$q_{i,1,1} = A(\frac{\lambda}{\mu_1})^i - \frac{\lambda+\mu_2}{\lambda+\alpha}\frac{\lambda+\mu_2}{\lambda+\mu_2-\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i+1}\left(1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-i-1}\right)q_{0,0,0}, i = 3,4,\ldots,K-1$ (11)

Choose A in such a way that (11) holds for $n = 2$ also. Then we get

$\mathrm{A} = \left(\frac{\mu 1}{\lambda}\right)^2\left\{P+\left(\frac{\lambda+\mu 2}{\lambda+\alpha}\right)\left(\frac{\lambda+\mu 2}{\lambda+\mu 2-\mu 1}\right)\left(\frac{\lambda}{\lambda+\mu 2}\right)^3\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-3}\right]\right\}$, where

$\mathrm{P} = \left(\frac{\lambda}{\mu 1}\right)\left\{\left(1+\frac{\lambda}{\mu 1}\right)\left[1-\left(\frac{\mu 2}{\lambda+\mu 2}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\right]\right]-\left(\frac{\mu 2}{\lambda+\alpha}\right)\left(\frac{\lambda}{\lambda+\mu 2}\right)^2\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-2}\right]-\left(\frac{\alpha}{\lambda+\alpha}\right)\right\}$. Substituting the value thus obtained for A in (11),

$q_{i,1,1} = \{A\left(\frac{\lambda}{\mu_1}\right)^i - \frac{\lambda+\mu_2}{\lambda+\alpha}\frac{\lambda+\mu_2}{\lambda+\mu_2-\mu_1}\left(\frac{\lambda}{\lambda+\mu_2}\right)^{i+1}\left(1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-i-1}\right)\}q_{0,0,0},\ i = 2,3,\ldots,K-1$ and $q_{K,1,1} = A(\frac{\lambda}{\mu_1})^K q_{0,0,0}$. To find the value of $q_{0,0,0}$, use the condition

$\sum_{i=0}^{K-1} q_{i,0,0} + \sum_{i=1}^{K} q_{i,1,1} + \sum_{i=2}^{K}\sum_{j=2}^{i} q_{i,j,2} = 1$. Then we have $q_{0,0,0} = \left(P_1 + P_2 + P_3 - P_4 + P_5 + P_6 + P_7 + P_8\right)^{-1}$ where $P_1 = \left(\frac{\lambda+\alpha}{\alpha}\right)\left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^{K}\right]$,

$P_2 = \left(\frac{\lambda}{\mu1}\right)\left[1 - \left(\frac{\mu2}{\lambda+\mu2}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)\left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\right]\right]$, $P_3 = A\left(\frac{\lambda}{\mu1}\right)\left(\frac{\lambda}{\mu1-\lambda}\right)\left[1 - \left(\frac{\lambda}{\mu1}\right)^{K-2}\right]$,

$P_4 = \left(\frac{\lambda}{\lambda+\alpha}\right)\left(\frac{1}{\lambda+\mu2-\mu1}\right)\left\{\left(\frac{\lambda^2}{\mu2}\right)\left[1 - \left(\frac{\lambda}{\lambda+\mu2}\right)^{K-2}\right] - \left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\left(\frac{(\lambda+\alpha)^2}{\mu2-\alpha}\right)\left[1 - \left(\frac{\lambda+\alpha}{\lambda+\mu2}\right)^{K-2}\right]\right\}$

$P_5 = \left(\frac{\lambda}{\mu1}\right)^{K} A$, $P_6 = \left(\frac{\lambda}{\lambda+\mu2}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)\left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^{K-2}\right]$, $P_7 = \left(\frac{\alpha\lambda}{\lambda+\alpha}\right)\left(\frac{\lambda+\mu2}{\alpha-\mu2}\right)\left(\frac{\lambda}{\lambda+\mu2}\right)^{2}\left\{\left(\frac{1}{\mu2}\right)\left[1 - \left(\frac{\lambda}{\lambda+\mu2}\right)^{K-3}\right] - \left(\frac{1}{\alpha}\right)\left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^{K-3}\right]\right\}$ and $P_8 = \left(\frac{\lambda}{\mu2}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}$.

3. SYSTEM PERFORMANCE MEASURES

In this section, we obtain some important system performance measures, which is need to find the optimal α value to activate the server.

Expected number of customers when the server is idle

$$E_{N0} = \sum_{i=1}^{K-1} i q_{i,0,0} = \left(\frac{\lambda+\alpha}{\alpha}\right)^{2}\left\{\left(\frac{\lambda}{\lambda+\alpha}\right) - K\left(\frac{\lambda}{\lambda+\alpha}\right)^{K} + (K-1)\left(\frac{\lambda}{\lambda+\alpha}\right)^{K+1}\right\} q_{0,0,0}$$

Probability that server is idle, $q_{.,0} = \sum_{i=0}^{K-1} q_{i,0,0} = \left(\frac{\lambda+\alpha}{\alpha}\right)\left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^{K}\right] q_{0,0,0}$

Probability distribution of the number of customers in the system under bulk service

$$q_{.,j,2} = \sum_{i=3}^{K} q_{i,j,2} + \sum_{i=2}^{K} q_{i,i,2} = \left\{\left(\frac{\lambda}{\lambda+\mu_2}\right)^{3}\left(\frac{\lambda+\mu_2}{\lambda+\alpha}\right)^{j}\left(\frac{\alpha}{\mu_2}\right)\left[1 - \left(\frac{\lambda}{\lambda+\mu_2}\right)^{K-2}\right] + \left(\frac{\lambda}{\lambda+\alpha}\right)\left(\frac{\lambda}{\lambda+\mu_2}\right)\left[1 - \left(\frac{\lambda}{\lambda+\alpha}\right)^{K-2}\right] + \left(\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\right\} q_{0,0,0}$$

Probability of customer loss from the system

$$P_l = q_{K11} + \sum_{j=2}^{K} q_{K,j,2} = \left\{\left(\frac{\lambda}{\mu_1}\right)^{K} A + \left(\frac{\alpha}{\alpha-\mu_2}\right)\left(\frac{\lambda+\mu_2}{\lambda+\alpha}\right)\left(\frac{\lambda}{\lambda+\mu_2}\right)^{K}\left[1 - \left(\frac{\lambda}{\lambda+\mu_2}\right)^{K-2}\right] + \left(\frac{\lambda}{\mu_2}\right)\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\right\} q_{0,0,0}$$

Expected Busy Period of the Server

Busy period of the server is defined as the duration of the time the server is continuously busy once it is activated and before the server next become idle. In this system, the server activation occurs at the time at which a random variable T realizes. Here T is an exponentially distributed random variable with parameter α. Once the server becomes active, it will provide batch service to all units waiting if there is at least two in the system and then turn in to single service for those who joined during the service time of the batch service. The server will continue single service until the system becomes empty and then goes for vacation. In this system, the expected time the server is busy is the sum of the expected time to reach from the state $(i+j, j, 2)$ to $(i, 1, 1)$ and from $(i, 1, 1)$ to state $(0, 0, 0)$. Here we use a recursive approach to compute the expected busy period. Define $T_{i,j,2}$ as the time to reach the state

$(i,1,1)$ from the state $(i+j,j,2)$ and $T_{i,1,1}$ as the time to reach the state $(0,0,0)$ form the state$(i,1,1)$. Then the expected busy period of the server is

$$E[B] = \sum_{i=1}^{K-2}\sum_{j=1}^{K-i} E\left[T_{i+j,i,2}\right] + \sum_{i=1}^{K} E\left[T_{i,1,1}\right]$$

Let W be the event that at the time of realization of T, there are j waiting customers in the system and A denote the event that i arrivals occurred during the service time of the batch for $j = 2,3,\ldots,K-i$ and $i = 1,2,\ldots K-2$. Then we can write

$$\sum_{i=1}^{K-2}\sum_{j=2}^{K-i} E(T_{i+j,j,2}) = \sum_{i=1}^{K-2}\sum_{j=2}^{K-i} E(T_{i+j,j,2} / W \text{ and } A) P(W \text{ and } A),\ K > 2$$

$$= \sum_{i=1}^{K-2}\sum_{j=2}^{K-i} \frac{1}{\mu_2} q_{j,0,0} \frac{\mu_2}{\lambda+\mu_2} \left(\frac{\lambda}{\lambda+\mu_2}\right)^i$$

$$= \frac{\lambda}{\alpha(\lambda+\mu_2)} \left\{\frac{\lambda}{\lambda+\alpha} \left(1-\left(\frac{\lambda}{\lambda+\mu_2}\right)^{K-2}\right. - \frac{\lambda}{\mu_2-\alpha}\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\left(1-\left(\frac{\lambda+\alpha}{\lambda+\mu_2}\right)^{K-2}\right)\right\} q_{0,0,0}$$

Proceeding as in S. M. Ross (2010), we can obtain

$\sum_{i=1}^{K} E(T_{i,1,1}) = \frac{K}{\mu_1-\lambda} - \frac{\lambda}{(\mu_1-\lambda)^2}\left(1-\left(\frac{\lambda}{\mu_1}\right)^K\right)$. Therefore, expected server busy period is

$$E(B) = \frac{\lambda}{\alpha(\lambda+\mu_2)} \left\{\frac{\lambda}{\lambda+\alpha}\left(1-\left(\frac{\lambda}{\lambda+\mu_2}\right)^{K-2}\right. - \frac{\lambda}{\mu_2-\alpha}\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\left(1-\left(\frac{\lambda+\alpha}{\lambda+\mu_2}\right)^{K-2}\right)\right\} q_{0,0,0} + \frac{K}{\mu_1-\lambda} - \frac{\lambda}{(\mu_1-\lambda)^2}\left(1-\left(\frac{\lambda}{\mu_1}\right)^K\right)$$

Expected idle time of the server

Idle period of the server is defined as the time at which the system becomes empty after the completion of all services to the time at which the server starts service next after the realization of the random variable T. Let I denote the idle period random variable. Then the expected idle period $E(I)$ can be computed as $E(I) = \sum_{i=1}^{K-1}\left(\frac{1}{\alpha}+\frac{i}{\lambda}\right)P(S_i < T < S_{i+1}) + \frac{K}{\lambda}P(T > S_K)$, where S_i is the time at which the ith arrival occurred. Then $P(S_i < T < S_{i+1}) = \frac{\alpha\lambda^{i-1}}{(\alpha+\lambda)^i}$ and $P(T > S_K) = (\lambda/(\lambda+\alpha))^K$. Then we have $E(I) = \frac{1}{\alpha}\left[1-\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\right] + \left(\frac{1}{\alpha}+\frac{1}{\lambda}\right)\left[1-(K-1)\left(\frac{\lambda}{\lambda+\alpha}\right)^K - K\left(\frac{\lambda}{\lambda+\alpha}\right)^{K-1}\right. + \frac{K}{\lambda}\left(\frac{\lambda}{\lambda+\alpha}\right)^K$

Expected cycle length Expected cycle length(EC) is $EC = EB + EI$

4. COST ANALYSIS

In this section, we construct a net profit function per cycle for the T-policy M/M/1/K queuing system with single/bulk service, in which T a decision variable. Our objective is to determine the optimum threshold α of T say α *, at minimum cost or maximum profit. Various system performance measures obtained based on assumed numerical values given to the system parameters. For that, we consider the following cost associated with the system.

Let C_1 be profit by way of server kept idle per cycle, C_2 be the cost of server activation per cycle, C_3 be waiting cost per cycle and C_4 be the holding cost of customers per unit time when server idle. Then the total expected cost per cycle,

$TEC = C_2\, E(B) + C_3\, (\frac{1}{\alpha}) + C_4 E_{N0}\, EC$. Then the net profit per cycle (NP) is

$NP\,(\alpha) = C_1\, EI - TEC.$

Due to the complex nature of the net profit function, it is difficult to establish the global optimal value of α. So we do numerical illustration with respect to net profit function for the given input parameters and computed it for various values of α. We can see that the net profit per cycle is maximum for the given input parameters.

4.1. Numerical Illustration

In this section, we provide certain numerical illustrations. For given input parameters, we compute various system characteristics and see that how the α value to start the server affects these characteristics. Various system performance measures are computed and their values and graphs are given in the following tables and figures.

For $\lambda = 2,\ \mu_1 = 3,\ \mu_2 = 5, K = 5, C_1 = 100, C_2 = 10, C_3 = 100$ and $C_4 = 1$, net profit per cycle is calculated and are given in Table 4.1. From the table it can be see that net profit per cycle increases first, reaches a maximum value 36.5486 for α=3.51 and then decreases as α increases.Hence α^*=3.51 is optimum here.

Table 1

α	1.51	2.01	2.51	3.01	3.51	4.01	4.51	5.01	5.51
NP	-0.68	25.98	34.28	36.54	36.62	35.88	34.86	33.79	32.76
α	6.01	6.51	7.01	7.51	8.01	8.51	9.01	9.51	10.01
NP	31.79	30.91	30.11	29.39	28.73	28.14	27.60	27.10	26.65

For $\lambda = 3, \mu_1 = 5, \mu_2 = 30, K = 5$, we have calculated the customer loss probability from the system and are given in Table 2. It shows that α increases, the customer loss probability decreases.

Table 2

α	0.1	0.15	0.2	0.25	0.3	0.35	0.4
Customer loss prob.	0.0032	0.0028	0.0023	0.002	0.0016	0.0014	0.0011

Table 3 gives the expected number of waiting customers when the server is idle for different values of α and for $\lambda = 2;\ \mu_1 = 3;\ \mu_2 = 15;\ \ K = 20$

Table 3

α	0.30	0.35	0.40	0.45	0.50	0.55	0.6	0.65	0.7
Expected no. of waiting customers when server idle	4.92	4.48	4.09	3.73	3.42	3.14	2.89	2.67	2.48
α	0.75	0.8	0.85	0.90	0.95	1.0	1.05	1.1	1.15
Expected no. of waiting customers when server idle	2.31	2.15	2.02	1.90	1.78	1.68	1.59	1.51	1.44

The next Table 4 shows the probability of an empty system for various values of α for the given parameter values $\lambda = 2;\ \mu_1 = 3;\ \mu_2 = 5;\ \ K = 10$

Table 4

α	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2
prob. of an empty system	0.32	0.28	0.25	0.23	0.21	0.19	0.18	0.17	0.15

α	1	1.5	2	2.5	3	3.5	4	4.5	5
Expected Waiting time of a customer	1	0.66	0.50	0.40	0.33	0.28	0.25	0.22	0.20

Expected busy period of the system is computed for various values of α and for the given parameter values $\lambda = 10,\ \mu_1 = 6, \mu_2 = 1$ and $K = 5$ and is given in Table 5. It can be seen that expected busy period increases for increasing value of alpha and expected idle time decreases for increasing value of α.

Table 5

α	1.5	3.5	5.5	7.5	9.5	11.5	13.5	15.5	17.5
Expected server busy period	6.14	6.15	6.16	6.16	6.16	6.16	6.16	6.16	6.16
Expected idle time of the server	2.09	1.56	1.36	1.27	1.21	1.17	1.15	1.13	1.11

5. CONCLUSION

In this paper, we have studied the optimal service management of an M/M/1/K queuing system with T-Policy which can provide single service and bulk service. We have studied the joint distribution of number of customers in the system, number of customers in service and state of the server. Various performance measures of the system are computed and performed numerical illustrations on the performance measures. We have observed numerically the optimal parameter value of the random variable under study. This paper can also be extended to general service time distributions.

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